Royster Chapter 1 PhL 2.1.05

At some point I will merge these short notes into the others, but for now separate so I can print and add.

1.1 History. In 1676 **Leibniz** thought about a geometry of position. Then in 1700's **Euler** and **Gauss** got on board. Position or location meaning points on some kind of surface (space). In 1847 **Listing** used the actual word topology in a book, but the accepted name at that time was analysis situs (of location). But it was Bernard **Riemann** (1826-1866) who thought about connectivity and holes and about spaces beyond R^3.

1.2 Sets and Set Operations The **roster method** is { 1,2,3}, the **set builder method** is $\{ x \in R \mid x < 5 \}$. Synonyms are **element, object, member**. Convention is sets are caps, elements are lower case. When you say $A \subset B$, what you really means is $A \subseteq B$ because the possibility of A = B is included. Intersection, union, etc.

A **universal set** is your frame of reference like "the reals".

Complement of A is written in several ways: $CA = A^c = A'$.

The notation A \ B means what is in A but not in B, so this is not complement "as such". This is really what you might mean by A - B. He calls A \ B by the name **set difference**. "A not in B". I think the complement follows, but he does not comment on this.

1.3 Product sets and theorems about such are stated.

1.4 Functions. When you write y = f(x), x is the **preimage** and y the **image**. The set of all such points are **domain** and **range**. (**codomain** = range). But then we re-use words so that f(A) = **image** for some set A in domain, and similarly $f^{-1}(B)$ would be the **inverse image** of B. **One to one** is **injective**, **onto** is **surjective**, and if you have both, you have a **bijection** and the inverse of f then exists. The identity and notion of a composite function are given. Def of a sequence.

1.5 Equivalence Relations. A relation for a set X has elements $\{ x_1, x_2 \}$ and is thus an element of X x X, a direct product set as we just learned above in 1.3 above. xRy means "x is related to y" in some way. In **equivalence relation** is a special case where we have xRx for all x, $xRy \Rightarrow yRx$ and finally $xRy, yRz \Rightarrow xRz$. This special relation has the notation **x ~ y**. The **equivalence class** denoted **[x]** of x is the set of all equivalent y and of course includes x.

1.6 Cardinality. Denumerable = countably infinite = countable. Else uncountable. Rationals can be counted, reals cannot.