

Problem Set 1

1. (Exercise 2, page 11) Prove that if A has precisely n distinct elements and B has precisely m distinct elements, where m and n are positive integers, then $A \times B$ has precisely mn distinct elements.
2. (Exercise 3, page 11) Let A and B be sets, both of which have at least two distinct members. Prove that there is a subset $W \subset A \times B$ that is not the product of a subset of A with a subset of B .
3. (Exercise 1, page 14) Let $f: A \rightarrow B$ be given. Prove the following:

- (a) For each subset $X \subset A$, $X \subset f^{-1}(f(X))$.
- (b) For each subset $Y \subset B$, $f(f^{-1}(Y)) \subset Y$.
- (c) If f is injective, then for each subset $X \subset A$,

$$f^{-1}(f(X)) = X.$$

- (d) If f is surjective, then for each subset $Y \subset B$,

$$f(f^{-1}(Y)) = Y.$$

4. (Exercise 4, page 14) Let $f: A \rightarrow B$ be given.
 - (c) If Y is a subset of B then $f^{-1}(B \setminus Y) = A \setminus (f^{-1}(Y))$.
 - (d) If $X \subset A$ and $Y \subset B$, then

$$f(X \cap f^{-1}(Y)) = f(X) \cap Y.$$

5. (Exercise 7, page 15) Let X be a set and $A, B, C \subset X$. The function $\chi_A: X \rightarrow \{0, 1\}$ defined by

$$\chi_A(x) = \begin{cases} 1 & \text{if } x \in A \\ 0 & \text{if } x \notin A \end{cases}$$

is called the *characteristic function* of A . Show

- (a) $\chi_{A \cap B} = \chi_A \cdot \chi_B$, where $\chi_A \cdot \chi_B(x) = \chi_A(x)\chi_B(x)$.
- (b) $\chi_{A \cup B} = \chi_A + \chi_B - \chi_{A \cap B}$ and find a similar expression for $\chi_{A \cup B \cup C}$.
6. Prove that $\emptyset \times B = \emptyset$ for each set B .
7. Give an example to show that $f(A \cap B)$ may not equal $f(A) \cap f(B)$. Show that equality holds if f is injective.