

Problem Set 3

1. (Exercise 6, page 46) Let a and b be distinct points in a metric space X . Prove that there are neighborhoods N_a and N_b of a and b respectively such that $N_a \cap N_b = \emptyset$.
2. Let $\{x_n\}$ be a sequence in the metric space X and assume that $\{x_n\}$ converges to $x \in X$. Prove that this limit is unique, *i.e.*, if $\lim x_n = x$ and $\lim x_n = L$ then $x = L$.
3. Show that a finite subset of a metric space has no limit points and is therefore a closed set.
4. Let (X, d) be a metric space with the discrete metric. Prove
 - (a) Every subset of X is open.
 - (b) Every subset of X is closed.
 - (c) No subset of X has a limit point.