

Problem Set 7

1. In the product space $X \times X$, the set $\Delta = \{(x, x) \mid x \in X\}$ is called the *diagonal*. Prove that a space X is Hausdorff if and only if the diagonal of $X \times X$ is a closed set.
2. Prove that if X is connected, then every quotient space of X is connected.
3. Prove that if X is compact, then every quotient space of X is compact.
4. Give an example of a Hausdorff space which has a quotient space that is not Hausdorff.