Paradox of the Day PhL 2.18.14

This concerns a factor of two confusion relating bipolar doc and lines doc. I kept thinking $K_1 = \xi_1$ but that we not correct. In fact, $K_1 = -2\xi_1$ and this is now all made clear in bipolar doc.

1. In my new bipolar document, I show that

$$\xi = (1/2)\ln\left(\frac{s_1^2}{s_2^2}\right) = \ln\left(\frac{s_1}{s_2}\right) \qquad (4.5)$$

and my potential solution is then simply

$$\varphi_t(\xi) = \xi = \ln\left(\frac{s_1}{s_2}\right)$$

and the problem this is the solution to is this

$$\nabla_t^2\varphi_t(x,y) = 0 \qquad \varphi_t(C_1) = \xi_1 \qquad \varphi_t(C_2) = \xi_2 \qquad (5.4.3)$$

because this matches the two boundary conditions ξ_1 on the right and ξ_2 on the left.

2. In lines doc (6.2.1) I claim that

$$s_2/s_1 = e^{-B} \qquad \Rightarrow \qquad \ln(s_2/s_1) = -B \ . \qquad (6.2.1)$$

and therefore

$$B = \ln\left(\frac{s_1}{s_2}\right)$$

Therefore, it must be true that $B = \xi$ and these are the exact same variable.

3. In lines doc, my "scaling boundary condition" is this:

$$\varphi_t(\mathbf{x}) \approx \ln(s_2^2/s_1^2) \quad \text{// limiting form as point } \mathbf{x} = (x,y) \text{ moves far from the conductors} \qquad (5.3.13)$$

where φ_t is a solution to this transverse problem,

$$\nabla_t^2\varphi_t(x,y) = 0 \qquad \varphi_t(C_1) = K_1 \qquad \varphi_t(C_2) = K_2 \qquad K_1 - K_2 = K \qquad (5.4.3)$$

Now I certainly can identify $K_1 = \xi_1$ and $K_2 = \xi_2$ so I have this problem:

$$\nabla_t^2\varphi_t(x,y) = 0 \qquad \varphi_t(C_1) = \xi_1 \qquad \varphi_t(C_2) = \xi_2 \qquad \xi_1 - \xi_2 = K \qquad (5.4.3)$$

4. Here then is the paradox. In the bipolar doc, I claim that the solution to (5.4.3) is this

$$\varphi_t = \xi = \ln\left(\frac{s_1}{s_2}\right)$$

whereas in my lines doc, I claim instead that the solution is this

$$\varphi_t(\mathbf{x}) \approx \ln(s_2^2/s_1^2) \quad \text{// limiting form as point } \mathbf{x} = (x,y) \text{ moves far from the conductors} \tag{5.3.13}$$

Somewhere there is a factor of 2 error going on!!!

Web data. Site http://web.mit.edu/6.013_book/www/chapter4/4.6.html says that

$$\Phi = \frac{-\lambda_i}{2\pi\epsilon_o} ln\, r_1 + \frac{\lambda_i}{2\pi\epsilon_o} ln\, r_2 = \frac{-\lambda_i}{2\pi\epsilon_o} ln\, \frac{r_1}{r_2} \tag{18}$$

where λ_i is the charge per unit length and where $r_1 = s_1$ etc.

My lines doc solution to this problem is this:

$$\varphi(x,y,z) = \frac{1}{4\pi\xi}\, q(z)\, \varphi_t(x,y) \tag{5.1.1}$$

$$\varphi_t(\mathbf{x}) \approx \ln(s_2^2/s_1^2)$$

so my lines doc solution is then

$$\varphi(x,y,z) = \frac{1}{4\pi\xi}\, q(z)\, \ln(s_2^2/s_1^2) = \frac{1}{2\pi\xi}\, q(z)\, \ln(s_2/s_1)$$

and this AGREES with the web quote.

5. In lines doc I have: " We then have from (6.1.2) and (6.2.1), "

$$\varphi_t(\mathbf{x}) = \ln(s_2^2/s_1^2) = 2\ln(s_2/s_1)$$

$$\varphi_t(C_1) = 2\ln(s_2/s_1)|^{C1} = -2B_1$$
$$\varphi_t(C_2) = 2\ln(s_2/s_1)|^{C2} = -2B_2\,. \tag{6.3.1}$$

It seems strange that I have those factors of 2 sitting there. So look back to the quoted equations:

$$\varphi_t(\mathbf{x}) = \ln(s_2^2/s_1^2)\,. \qquad \text{for } \textit{all} \text{ values of } \mathbf{r}\text{, close and far} \tag{6.1.2}$$

$s_2/s_1 = e^{-B}$ => $\ln(s_2/s_1) = -B$. (6.2.1)

So the factors of 2 are just coming from $\varphi_t(\mathbf{x}) = 2\ln(s_2/s_1)$. So I basically have

$\varphi_t(C_1) = K_1 = -2B_1 = -2\xi_1$
$\varphi_t(C_2) = K_2 = -2B_2 = -2\xi_2$

When I wrote things up, I just "made up" the names K_1 and K_2. I made no claim they were equal to anything in particular. I later found out that $K_1 = -2\xi_1$, for example.