

Hi Dan,

6 March 2011

While fumbling around with Mehler transforms, I think I spotted an obscure errata in your Differential Equations book, 3rd Edition 1997. I don't see it in your website errata list last updated Nov 22, 2000.

In Section 78 page 351 you have

78. Integral Transforms: Infinite Intervals*

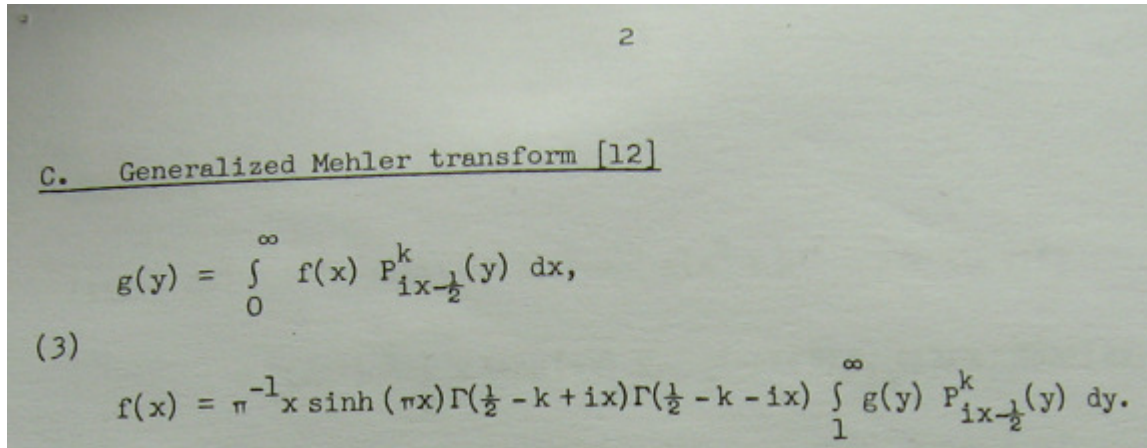
351

Mehler-Fock transform of order m , (see Sneddon [22, Chapter 7])

$$v(\xi) = \int_0^\infty \sinh(x) P_{i\xi-1/2}^m(\cosh x) u(x) dx,$$

$$u(x) = \int_0^\infty \xi \tanh(\pi\xi) P_{i\xi-1/2}^m(\cosh x) v(\xi) d\xi.$$

I think this is only correct for $m = 0$, but I don't have Sneddon's book to check. However, right now I am staring at page 2 of F. Oberhettinger and T. P. Higgins, *Tables of Lebedev, Mehler and Generalized Mehler Transforms*, which is reference [21] on your page 355, and here is what I see:



2

C. Generalized Mehler transform [12]

$$(3) \quad g(y) = \int_0^\infty f(x) P_{ix-\frac{1}{2}}^k(y) dx,$$

$$f(x) = \pi^{-1} x \sinh(\pi x) \Gamma(\tfrac{1}{2} - k + ix) \Gamma(\tfrac{1}{2} - k - ix) \int_1^\infty g(y) P_{ix-\frac{1}{2}}^k(y) dy.$$

In something I wrote a while ago, I presented the transform in this manner:

The generalized Mehler-Fock (Mehler-Fok) transform (Oberhettinger and Higgins* page 2) is this,

$$g(y) = \int_0^\infty d\tau P_{-1/2+i\tau}^m(y) f(\tau) \quad // \text{ expansion}$$

$$f(\tau) = (\tau/\pi) \operatorname{sh}(\pi\tau) \Gamma(1/2-m+i\tau) \Gamma(1/2-m-i\tau) \int_1^\infty dy P_{-1/2+i\tau}^m(y) g(y) \quad // \text{ projection}$$

and for $m = 0$, using $\Gamma(1/2+i\tau)\Gamma(1/2-i\tau) = \pi/\cosh(\pi\tau)$, one obtains the regular Mehler-Fock transform,

$$g(y) = \int_0^\infty d\tau P_{-1/2+i\tau}(y) f(\tau) \quad // \text{ expansion}$$

$$f(\tau) = \tau \operatorname{th}(\pi\tau) \int_1^\infty dy P_{-1/2+i\tau}(y) g(y) \quad // \text{ projection}$$

which can also be written as

$$G(\xi) = \int_0^\infty d\tau P_{-1/2+i\tau}(\operatorname{ch}\xi) f(\tau) \quad // \text{ expansion}$$

$$f(\tau) = \tau \operatorname{th}(\pi\tau) \int_0^\infty d\xi \operatorname{sh}\xi P_{-1/2+i\tau}(\operatorname{ch}\xi) G(\xi) \quad // \text{ projection}$$

* F. Oberhettinger and T. P. Higgins, *Tables of Lebedev, Mehler and Generalized Mehler Transforms*, (Boeing Scientific Research Laboratories, Mathematical No. 246, 1961). A 48 page monograph with about 16 transforms per page. The first author is a member of A. Erdelyi *et. al* mentioned above. We obtained this book by interlibrary loan from the Coe library at the University of Wyoming.

As another piece of evidence, here is a clip from your page 355 Nasim reference [19] :

If

$$f(y) = \frac{i}{2\pi} \int_{\sigma-i\infty}^{\sigma+i\infty} t \sin(\pi t) \Gamma\left(\frac{1}{2} - m - t\right) \Gamma\left(\frac{1}{2} - m + t\right) P_{-\frac{1}{2}+t}^m(y) F(t) dt, \quad (3.9)$$

then

$$F(t) = \int_1^\infty f(y) P_{-\frac{1}{2}+t}^m(y) dy \quad (3.10)$$

You can set $\sigma = 0$ and $t = ix$ to get something equivalent to Oberhettinger and Higgins.

Best regards,

Phil Lucht
Salt Lake City