This is installed, do not edit.

Appendix B. Summary of the three Euler substitutions

Substitution A $\quad t \pm \sqrt{c}\, x = \sqrt{a+bx+cx^2}$

$$\int^{x} dx\, \mathcal{R}(x,\sqrt{a+bx+cx^2}) = \int^{t(x)} dt\; 2\,\frac{-\sqrt{c}\,t^2 + bt - a\sqrt{c}}{(b - 2\sqrt{c}\,t)^2} * \mathcal{R}\left(\frac{t^2-a}{b - 2\sqrt{c}\,t}, \frac{-\sqrt{c}\,t^2 + bt - \sqrt{c}\,a}{b - 2\sqrt{c}\,t}\right)$$

where $t(x) = \sqrt{a+bx+cx^2} - \sqrt{c}\,x$. *(4.7)*

$$\int^{x} dx\, \mathcal{R}(x,\sqrt{a+bx+cx^2}) = \int^{t(x)} dt\; 2\,\frac{\sqrt{c}\,t^2 + bt + a\sqrt{c}}{(b + 2\sqrt{c}\,t)^2} * \mathcal{R}\left(\frac{t^2-a}{b + 2\sqrt{c}\,t}, \frac{\sqrt{c}\,t^2 + bt + \sqrt{c}\,a}{b + 2\sqrt{c}\,t}\right)$$

where $t(x) = \sqrt{a+bx+cx^2} + \sqrt{c}\,x$. *(4.8)*

Substitution B $\quad xt \pm \sqrt{a} = \sqrt{a+bx+cx^2}$

$$\int^{x} dx\, \mathcal{R}(x,\sqrt{a+bx+cx^2}) = \int^{t(x)} dt\; 2\,\frac{\sqrt{a}\,t^2 - bt + \sqrt{a}\,c}{(c-t^2)^2} * \mathcal{R}\left(\frac{2\sqrt{a}\,t - b}{c-t^2}, \frac{\sqrt{a}\,t^2 - bt + \sqrt{a}\,c}{c-t^2}\right)$$

where $t(x) = (\sqrt{a+bx+cx^2} - \sqrt{a}) / x$. *(5.7)*

$$\int^{x} dx\, \mathcal{R}(x,\sqrt{a+bx+cx^2}) = \int^{t(x)} dt\; 2\,\frac{-\sqrt{a}\,t^2 - bt - \sqrt{a}\,c}{(c-t^2)^2} * \mathcal{R}\left(\frac{-2\sqrt{a}\,t - b}{c-t^2}, \frac{-\sqrt{a}\,t^2 - bt - \sqrt{a}\,c}{c-t^2}\right)$$

where $t(x) = (\sqrt{a+bx+cx^2} + \sqrt{a}) / x$. *(5.8)*

Substitution C : $\quad t = \sqrt{c}\,\left(\sqrt{\frac{x-\beta}{x-\alpha}}\right)^{\pm 1}$ where $a+bx+cx^2 = c(x-\alpha)(x-\beta)$

$$\int^{x} dx\, \mathcal{R}(x,\sqrt{a+bx+cx^2}) = \int^{t(x)} dt\; 2\,\frac{c(\beta-\alpha)t}{(t^2-c)^2} * \mathcal{R}\left(\frac{\alpha t^2 - c\beta}{t^2-c}, \frac{c(\alpha-\beta)t}{t^2-c}\right)$$

where $t(x) = \sqrt{a+bx+cx^2} / (x-\alpha) = \sqrt{R} /(x-\alpha)$. *(6.7)*

$$\int^{x} dx\, \mathcal{R}(x,\sqrt{a+bx+cx^2}) = \int^{t(x)} dt\; 2\,\frac{c(\alpha-\beta)t}{(t^2-c)^2} * \mathcal{R}\left(\frac{\beta t^2 - c\alpha}{t^2-c}, \frac{c(\beta-\alpha)t}{t^2-c}\right)$$ *(6.8)*

where $t(x) = \sqrt{a+bx+cx^2} / (x-\beta) = \sqrt{R} /(x-\beta)$.