Edit Log PhL 11.5.16

I threw this paper together over the last 2 days, preceded by maybe 3 days of work on the topic. Right now I have a reasonable first cut, with eq nums and references.

I think now would be a good time to look at other references, so here we go:

PBM: This is the big Russian integral book series. My copies are in Russian. Page 66 of the indefinite integrals volume has relevant integrals. But there are no method comments in that section. There is no teaching in this volume, so I will continue to ignore it. If they had a huge section on Euler, I would want to cite it.

Dwight does not have *any* of my kind of integrals!! I am amazed at that. The Petit thing is too old. Zwillinger has nothing. So nothing I already have talks about this stuff.

Now go to the web a bit.

I downloaded the 1969 Piskunov book in English, and it mentions the Euler stuff at about the same level as in GR7. Added that to refs with early mention in my intro.

OK, I now feel I have done some due diligence on this topic.

I just did a quick pagination, came out pretty well I think. Document is then 20 pages.

Spelling check: done

I see nothing at Researchgate on this subject.

Eq num check:

It is now 2 PM, I will take a break and then do a final proofing.

Proofing pass starting 4PM 11/5/16

Title, note, TOC	ok
1. Introduction	ok
2. Comments about $R = a + bx + cx^2$	ok
3. A substitution that does not work (added picture)	ok
4. Method A	ok
5. Method B	ok
6. Method C	ok
7. More forms for the integral of $R^{-1/2}$	ok
References checked links	

I did more cleanup and then did a PDF and a release to xmission! All done! (ha!)

Update 11.6.16

After a night's sleep, I decided the introduction needs improving and I should replace the word method with the word substitution.

What do I know about the labels 1,2,3 for the subs?

GR7:

1. $\sqrt{a+bx+cx^2} = xt \pm \sqrt{a}$ for $a > 0$;
2. $\sqrt{a+bx+cx^2} = t \pm x\sqrt{c}$ for $c > 0$;
3. $\sqrt{c\,(x-x_1)\,(x-x_2)} = t\,(x-x_1)$ when x_1 and x_2 are real roots of the equation $a+bx+cx^2 = 0$.

Piskunov :

1. *First Euler substitution.* If $a>0$, then we put

$$\sqrt{ax^2+bx+c} = \pm\sqrt{a}x + t.$$

This is my A

2. *Second Euler substitution.* If $c>0$, we put

$$\sqrt{ax^2+bx+c} = xt \pm \sqrt{c};$$

This is my B

3. *Third Euler substitution.* Let α and β be the real roots of the trinomial ax^2+bx+c. We put

$$\sqrt{ax^2+bx+c} = (x-\alpha)\,t.$$

This is my C

OK, I have now installed a new Intro, there are all new eq nums, so go through rest of doc and update x refs to these. DONE, only one to do really.

Now replace method by substitution in certain places: done.

Pagination: Only changes in sections 1,2,3.

New proofing of the entire document:

title and TOC	OK	
Section 1	OK	
Section 2	OK	
Section 3	OK	
Section 4	OK	
Section 5	OK	
Section 6	OK	
Section 7	OK	
Appendix	OK	
References	OK	now 2:45 PM !!!

Checked all equation periods. No thrown eq nums.

Time for a PDF. OK, except (1.6) eq num is tabbed too far to the left. I tried to fix it and it is still the same. On Black if I zoom to 150% I see the indentation problem, but not at 125%. OK, trying another PDF. This time OK. Will now do a new release to Xmission.

Update Mon 11.7.16 and 11.8.16

After another night, want to do two things.

(1) see if the Goldstein sign error is really a PHL error

(2) look harder at what Euler does early in his paper. Use the German version.

(1) Here is the integral in question,

$$\int^{x} dx\, \frac{1}{\sqrt{R}} \quad \doteq \quad -\frac{1}{\sqrt{-c}} \cos^{-1}\left(\frac{-2cx - b}{\sqrt{b^2-4ac}}\right)$$

What is the significance of the leading minus sign. Write the above as (and track the sign in red)

$$I = -\frac{1}{\sqrt{-c}} \cos^{-1}\left(\frac{-2cx - b}{\sqrt{b^2-4ac}}\right) + K$$

where the K is there if you want a real integral between some x_0 and x. Now

$$-\sqrt{-c}\, I = \cos^{-1}\left(\frac{-2cx - b}{\sqrt{b^2-4ac}}\right) - \sqrt{-c}\, K$$

$$-\sqrt{-c}\, (I + K) = \cos^{-1}\left(\frac{-2cx - b}{\sqrt{b^2-4ac}}\right)$$

Now take cosine of both sides to get

$$\cos[-\sqrt{-c}\, (I+ K)] = \frac{-2cx - b}{\sqrt{b^2-4ac}}$$

The minus sign makes no difference because the above is the same as

$$\cos[+\sqrt{-c}\, (I+ K)] = \frac{-2cx - b}{\sqrt{b^2-4ac}}$$

Conclusions:

(1) the integral really has a minus sign and you cannot throw that sign out.

(2) Suppose you have an equation

$$A = - B$$

The sign is meaningful at this point. But when you take the cosine of both sides, the sign no longer matters because cos is an even function

$$\cos(A) = \cos(-B) = \cos(+B)$$

More generally suppose you have

$$f(x) = - g(x)$$

where the sign is completely meaningful. Now suppose E(x) is some even function in x. Then

$$E[f(x)] = E[-g(x)] = E[g(x)]$$

and after doing that, the sign no longer matters.

Conclusion: The integral *really must have the minus sign*. Otherwise diff(RHS, x) will give - $1/\sqrt{R}$
OK, I am happy with that issue

(2) The Euler paper.

OK as of 8 PM I have rewritten my Appendix A and it is more honest and accurate, and I think more interesting now. I will proof it tomorrow, install and then release once again.

Update Mon 11.9.16

Read the PDF after it cooled overnight, have a whole batch of errata which are described in that file. I will do the edits and re-release. Now is the right time to get out the bugs.

quaeramus integrali formulae α

$b^2/(4ac)$ b^2 4ac

Did two major things in addition to small errata
(1) improved the analytic continuation discussion of Section 7
(2) corrected and improved the Euler paper discussion in Appendix A.

Update Mon 11.10.16

Well, another light review uncovered still more errata, so here we go with yet another release. Did a lot of cycles but now have a new release to Xmission.

Update Mon 11.11.16

I did a pretty careful read through of the PDF on Xmission and could not detect a single error (though there surely are still some). I did not do all the x(t) type algebra this reading, but I am pretty confident it is all done correctly. I don't have a source to check my final results.

This thing I feel is now ready for Researchgate, so I will release it right now. It will need an abstract.

Abstract

The Euler Substitutions are used to integrate rational functions of x and $(a+bx+cx^2)^{(1/2)}$. In this document, appropriate for any calculus student, details of these substitutions are worked out and the results are stated in a systematic manner. As an example, the indefinite integral of $(a+bx+cx^2)^{(-1/2)}$ is computed using all the Euler Substitutions and is then expressed in twelve different ways. An Appendix reviews Euler's original 1786 Russia-published paper which is written in Latin.

DONE (for now). Then made a Maple and Visio index, DONE (very short).

This paper is 27 pages long, surely the most detailed treatment of this subject in existence.