

Hi Dan,

I would like to report two errata for GR7 (my list grows very slowly).

integral in question: GR7 page 317 item 3.197.2¹¹

Here is what GR7 shows:

$$2.11 \quad \int_u^\infty x^{-\lambda} (x + \beta)^\nu (x - u)^{\mu-1} dx = u^{-\lambda} (\beta + u)^{\mu+\nu} B(\lambda - \mu - \nu, \mu) {}_2F_1 \left(\lambda, \mu; \lambda - \mu; -\frac{\beta}{u} \right) \\ \left[\left| \arg \frac{u}{\beta} \right| < \pi \text{ or } \left| \frac{\beta}{u} \right| < 1, \quad 0 < \operatorname{Re} \mu < \operatorname{Re}(\lambda - \nu) \right] \quad \text{ET II 201(8)}$$

There are two errors in this formula.

- the c parameter of F(a,b;c;z) should be $\lambda - \nu$ and not $\lambda - \mu$
- the first condition should be $|\arg(\beta/u)| < \pi$ and not $|\arg(u/\beta)| < \pi$

The ET II 201 (8) integral quoted appears on page 146 of my Russian edition as follows:

	$f(x)$	$[\Gamma(\mu)]^{-1} \int_y^\infty f(x) (x-y)^{\mu-1} dx = h(y; \mu)$
(8)	$x^{-\lambda} (x + \alpha)^\nu, \\ 0 < \operatorname{Re} \mu < \operatorname{Re}(\lambda - \nu)$	$y^{\mu+\nu-\lambda} \frac{\Gamma(\lambda - \mu - \nu)}{\Gamma(\lambda - \nu)} \times \\ \times {}_2F_1(-\nu, \lambda - \mu - \nu; \lambda - \nu; -\alpha/y), \\ \arg(\alpha/y) < \pi \text{ или } \alpha/y < 1$

and one can make the substitutions $\alpha \rightarrow \beta$, $y \rightarrow u$ to make the comparison. Also needed is the first Kummer relation say from HT I page 105, which does not affect the c or z arguments of F(a,b;c;z),

$$(1) \quad u_1 = F(a, b; c; z)$$

$$(2) \quad = (1 - z)^{c-a-b} F(c - a, c - b; c; z)$$

$$\Rightarrow F(-\nu, \lambda - \mu - \nu; \lambda - \nu; -\alpha/y) = y^{-\nu-\mu} (y + \alpha)^{\mu+\nu} F(\lambda, \mu; \lambda - \nu; -\alpha/y)$$

Before I found this thing in ET II, I derived it from scratch using the fundamental integral representation for F(a,b;c;z) and some variable changes. Here is the sequence of events starting with HT I p 59 (10)

2.1.3. The fundamental integral representations

If $\operatorname{Re} c > \operatorname{Re} b > 0$, we have Euler's formula

$$(10) \quad F(a, b; c; z) = \Gamma(c) [\Gamma(b) \Gamma(c - b)]^{-1} \int_0^1 t^{b-1} (1 - t)^{c-b-1} (1 - tz)^{-a} dt.$$

$$\int_0^1 dt \, t^{b-1} (1-tz)^{-a} (1-t)^{c-b-1} = B(b, c-b) F(a, b; c; z) \quad // \text{ do } x = 1/t :$$

$$\int_1^\infty dx \, x^{a-c} (x-z)^{-a} (x-1)^{c-b-1} = B(b, c-b) F(a, b; c; z) \quad // \text{ agrees HT I p115 (6); do } y = ux:$$

$$\int_u^\infty dy \, y^{a-c} (y-uz)^{-a} (y-u)^{c-b-1} = u^{-b} B(b, c-b) F(a, b; c; z) \quad // \text{ let } z = -\beta/u:$$

$$\int_u^\infty dy \, y^{a-c} (y+\beta)^{-a} (y-u)^{c-b-1} = u^{-b} B(b, c-b) F(a, b; c; -\beta/u) \quad // a=-v, b=\lambda-\mu-v, c=\lambda-v :$$

$$\int_u^\infty dx \, x^{-\lambda} (x+\beta)^v (x-u)^{\mu-1} = u^{\lambda+\mu+v} B(\lambda-\mu-v, \mu) F(-v, \lambda-\mu-v; \lambda-v; -\beta/u) \quad // \text{ as in GR4 p 286}$$

$$F(-v, \lambda-\mu-v; \lambda-v; -\alpha/u) = u^{-v-\mu} (y+\alpha)^{\mu+v} F(\lambda, \mu; \lambda-v; -\alpha/u) \quad // \text{ Kummer}$$

$$\int_u^\infty dx \, x^{-\lambda} (x+\beta)^v (x-u)^{\mu-1} = u^{-\lambda} (\beta+u)^{\mu+v} B(\lambda-\mu-v, \mu) F(\lambda, \mu; \lambda-v; -\beta/u) \quad // \text{ corrected GR7 p 317}$$

I see from the superscript 3.197.2¹¹ that this integral has had some "history" in GR.

Best regards,
Phil Lucht
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P.S. I don't see this in your errata PDF dated April 11, 2008 located at <http://www.mathtable.com/gr>.

P.P.S. The above integral arose when I was doing some Mehler transform integrals. It seems odd to me that these relatively simple (to state, not always to do) integrals do not appear in GR7, though some of this form appear on page 788. For example, these do not appear in GR7 :

$$\int_0^\infty d\tau \, P_{-1/2+i\tau}(y) \cos(a\tau)/\cosh(\pi\tau) = (1/\sqrt{2}) \, 1/\sqrt{y+cha}$$

$$\int_0^\infty d\tau \, P_{-1/2+i\tau}(y) \cos(a\tau) / \cosh^2(\pi\tau) = (\sqrt{2}/\pi) \, (1/\sqrt{y-cha}) \tan^{-1}[\sqrt{y-cha}/\sqrt{1+cha}]$$

$$\int_0^\infty d\tau \, P_{-1/2+i\tau}(y) \operatorname{ch}(b\tau) / \cosh^2(\pi\tau) = (\sqrt{2}/\pi) \, (1/\sqrt{y-\cos b}) \cot^{-1}[\sqrt{1+\cos b}/\sqrt{y-\cos b}] \quad |b| < 2\pi$$

These and many others appear in PBM 2003 Special Functions vol 3 and/or in Boeing Report 246 of Oberhettinger and Higgins. The $P_{-1/2+i\tau}(y)$ appear in both spherical and toroidal harmonics.