Derivation of (3.7)

Look at (2.4). The functions of the vector Φ are the ortho polys ϕ defined by interval (a.b) and weight w(x). As we increase N, the ϕ don't change, there are just more of them in the matrix equation.

So the matrix equation at large but finite N says this:

$$\lambda_N \Phi_N = T_N \Phi_N + r_N \qquad (1)$$

The same equation at infinite N says this

$$\lambda_\infty \Phi_\infty = T_\infty \Phi_\infty$$

and $r_\infty = 0$. This last is an ∞ x ∞ matrix. If we look at the upper left $(N+1)^2$ corner of this matrix we see

$$\lambda_\infty \Phi_N = T_N \Phi_N \qquad (2)$$

Notice that the components of Φ don't change as noted above, nor does the matrix T_N since it is composed of the recursion coefficients which do not in turn change.

So we can now compute (1) - (2) to get

$$(\lambda_N - \lambda_\infty)\mathbf{\Phi_N} = \mathbf{r_N}$$

and now dot both sides with Φ_N to get

$$(\lambda_N - \lambda_\infty)\mathbf{\Phi_N} \bullet \mathbf{\Phi_N} = \mathbf{\Phi_N} \bullet \mathbf{r_N}$$

which says

$$(\lambda_N - \lambda_\infty)\Sigma_{n=0}^{N} \phi_n^2 = \alpha_N \phi_{N+1} \phi_N$$

where we use (2.7) for r, knowing that $\phi_{-1} = 0$ (in 3.5, we see that saying the x_i are zeros of $J_\upsilon(x_i)$ causes the ϕ_{-1} term to be 0). Thus we get

$$\lambda_N = \lambda_\infty + \alpha_N \phi_{N+1} \phi_N / (\Sigma_{n=0}^{N} \phi_n^2) \qquad \text{QED}$$

which is (3.7). Notice that he says that λ_i are the eigenvalues of T_N, so he identifies $\lambda_i = \lambda_\infty$. Then the finite but large eigenvalues are $\lambda_N = 1/x_i^2$.