Paradox 143.56 10.7.16

1. On the one hand:

Given a function f(**r**) on E^{N-1} and no constraints, a **stationary point** (sometimes called a **critical point**) is a value of **r** for which $\nabla^{(N-1)} f(\mathbf{r}) = \mathbf{0}$.

2. On the other hand:

Let **r** be a stationary point for f(**r**) where f(**r**) = K. The surface f(**r**) = K has a normal ∇f(**r**) which is a vector in E^N. No normal vector can be a null vector because it is the normal vector!

3. Paradox. Item one say ∇f(**r**) = **0** at a stationary point, item two says this is impossible.

Idea A. Look at an example of 1. Think z = f(x,y) which is g(x,y,z) = z - f(x,y) = 0. There is a 2D normal ∇f which can vanish at the north pole of a surface z = f(x,y). But the 3D gradient of g is this: $\nabla g = \hat{\mathbf{z}}$ and this is not 0. So maybe you need to be clearer about which gradient you are talking about. The surface f(**r**) = K is a surface of dimension N-1 in E^N. The object ∇f is a vector in E^N. I think it is true that this vector is never null.

I think my item 1 above is wrong.

Let f(**r**) be a function on E^N. Let (**r**, u) be a point in E^{N+1} where u = f(**r**) is regarded as the $(N+1)^{th}$ coordinate. Define g(**r**,u) ≡ u- f(**r**). The equation g(**r**,u) = 0 describes a surface in E^{N+1}. This surface has a normal $\nabla^{(N+1)} g(\mathbf{r},u)$ which is a vector in E^{N+1}. This gradient is never null for a smooth surface. A critical point of the function f(**r**) is one where $\nabla^{(N)} f(\mathbf{r}) = 0$. This gradient is null at a stationary point of the function f(**r**).

Given a function f(**r**) and no constraints, a **stationary point** (sometimes called a **critical point**) is a value of **r** for which ∇f(**r**) = **0**.