Edit Log for new Lagrange doc PhL 10.8.16

History of New Lagrange Doc

On Sept 16, 2016 I started a review of Goldstein after a long technical break. I got to the virtual displacement stuff and decided it was time to get that nailed down. Found paper of the two Indians on this subject.

Somehow this led me to look at my Lag Mult paper, maybe thinking about the parallel between my doc and Goldstein's stuff on multipliers. How can I claim to have a good Method Of Lag Mult document if I *don't even mention* this classic application of multipliers? So I wanted to add that stuff. I had only a passing mention in a certain Comment 4 which was very hazy.

As I reviewed Lag doc, I felt my matrix derivation was weak on the notion of "eliminating variables". This caused me On Sept 20 to write Appendix C to clarify this, and after doing so I felt that my proof weakness was at least somewhat repaired. Since that was my *main* proof, it was important that it be solid.

Sept 23 I started App D based on the Goldstein and Indians stuff and worked on that for a few days. I then resumed Goldstein review Sept 29. Read about solving non-linear ODE's and got this into App D. Finished Goldstein Ch 2 and then started into a formal main body of Lagrange doc update cycle in the old folder.

BUT, on Oct 5 I found that my logic was lousy, the geometric proof should be the main act, and I pulled my doc from Xmission and from Researchgate, leaving there just a dummy abstract.

I needed a major rework. I commented out the doc in my index.html file, learned how to do that. So I then spent the next 20 days (Oct 5 to Oct 25) rewriting Lag doc and today I released it. Went from 64 to 116 pages, I think it is much better now.

My work on this Lag doc update started Sept 20 I would say, and ended Oct 25, so maybe 35 days so we can call that 5 weeks of work just to do an update! So it goes, the paper is now "pretty good".

So far I have written Section 1 of New Lag Doc. This obtains the key results using the notions of surfaces and tangent space and perp space. No matrices are involved, I think the development is bulletproof pretty much. This I now think is the "right way" to obtain the results.

In Section 2 I quote the "official method" which gives these same results.

Sun 10.9.16.

I have added both my sphere examples which now comprise a new Chapter 3. I will now undertake the massive problem of writing the equation numbers for my new Sections 1,2 and 3.

Section 1:

add eq nums and format: done
complete proof/edit pass: done

Section 2

add eq nums and format: done
complete proof/edit pass: done

The above took me quite a while, there is a lot of detail.

Tues Oct 11 continue

No work for 2 days, will read what is there before adding more.

Section 1. Read through and made many changes. Trying to make each idea as clear and unambiguous as possible, and to allow as there are degenerate cases which are being ignored, and so on. Clean definitions are needed here. I think it is better now and will start into Section 2.

Section 2. Read through and made many changes with same goal as the above. Done.

Section 3. Updated all eq nums, edited the text, then added a third generalized sphere example.

Here are my proposed next two sections

4. Example 4: Extremal distances between an ellipse and a circle

5. Example 5: The Boltzmann factor in Statistical Mechanics

Luckily the example numbers match the example numbers!

Section 4: All entered and I defined the f_i notations so need make only tiny changes. All done!

Section 5: Entered, now I will read it all. I added a little intro.
5.1 is OK
5.2 is OK
running into some trouble here.

Resume 10.13.16

I had to digress and update my bins and balls document so equations appearing on wiki and elsewhere make sense. I am now starting over on my review of Section 5

Section 5.1 OK
Section 5.2 very good, I like it. High level of clarity and interest I think.
Section 5.3 is much better now, I have the narrow peak nailed I think.
Section 5.4

Ouch. I still have the peak width calculation wrong. It is so confusing now that it gets a separate doc and I will do that next. Maybe my original calculation was done correctly and my change to that was wrong.

Resume 10.14.16

I have the math cleaned up about the peak calculation. But now I have a new problem. I keep saying that the peak should be super narrow, but I don't really know what situation I am referring to. If it is atoms of a gas in a box, then the energy states are those where $\varepsilon = (1/2)mv^2$ and you end up with a state distribution N_i which is then N(v) which is "the Maxwell distribution" and which is NOT strongly peaked at all, so my claim does not apply to an ideal gas. It would apply to a system where there are a small number of quantum states but I don't know what kind of system that would be for large particle count. Maybe a box of particles having spin and you are interested only in the spin energy stuff.

I have to go off and ponder all this stuff before finalizing my Boltzmann Example.

Resume 10.15.16

Got a log of detail work done yesterday on the box of gas problem, but am confused about the variance issue, so going on that now at 6:30AM. Another thing is to redo like this:

Status 10.18.16

I have done a lot. Much was building for my Maxwell distribution section which is not started yet. The rest is getting the 3-State Example where I want it to be. I think it is finally there now. I will proof the whole Section 5 right now.

Section 5

Opening preview: OK

5.1 Statement of The Boltzmann Extremum Problem

Excellent down to (5.1.4), providing physics background for the problem

The rest is just fine too, a good section.

5.2 Solution of The Boltzmann Extremum Problem

OK, I obtain a complete solution of the problem.

5.3 More details of the solution.

Made fixes, it is very good I think. Had a momentary doubt about half width but now think it is all OK.

5.4 Example: N particles in 3 states : a numerical example

OK here, had a temporary worry about half width, but now OK.

I will now start on the Maxwell 5.5 in a separate doc.

Status 10.19.16

I started into Section 5.5 yesterday. Today I revamped Section 5.3 by adding much more no the meaning of absolute temperature that the equation $\beta = 1/(kT)$. I show how T is defined, and then how k can be measured and that β has units of 1/energy. This seemed a good place to say all this stuff, and no equations had to be added! I lost my mws file with the helium constants, will have to rebuild it.

I next made another attempt to explain my "very narrow peak" idea for an ensemble of a billion systems. This has been a problem now for several weeks, it evades clarity, but I think I have nailed it down now at the end of Section 5.3.

So OK, time to do yet another read through of Sections 5.1 to 5.4 to have that cleanly in mind while IO write section 5.5. I know of no other way to do this. I am not doing details except for sections where changes have been made, especially qualitative ones.

5.1 done
5.2 done
5.3 done
5.4 done

I then went on to write a big v2 chunk of 5.5 including the quantum mechanics stuff. So after a slow start, I got a lot done today, and it is now 9:15 PM. The debate ended at 9 and I recorded it.

Status 10.20.16

I have finished Section 5.5 and have added eq nums. Time for a new proofing and then add a summary at the end.

Section 5.4

The System	OK
Connection To Sections 5.2 and 5.3	OK
The Maxwell-Boltzmann speed distribution	OK
Mean Values	OK
Determination of the derived Lagrange Multiplier A	OK
Quantum Mechanics and Determination of V	OK

Status 10.21.16

I had planned to add more to Section 5.5 about $N_{nx,ny,nz}$ and $\mathcal{N}(\mathbf{v})d^3v$ but after writing it up, I decided it was not interesting at all. It is now in a separate doc. So once again I think I am done with 5.5, but I spent all day on this digression.

The question now: do I want to say anything at all about matrices and their rank? My original Lagrange doc was just terrible. It introduces the R matrix, then I prove the theorem that if df = 0, the rows are linearly independent. and so R falls below full rank. Bucks got me into that subject really. But now the proof is totally trivial: if the gradients are linearly independent, the R matrix rank falls below full rank, there is nothing to prove, it is just plain stupid!

In that doc I was trying *not* to start with the fact that the gradients line up and I wanted that to be a conclusion, something I was "proving" with all the matrix work. Here is the logic flow of old doc:

1. Define the R matrix.
2. Theorem 1 with *very long proof*: if df = 0 with constraints, then det(R) < full rank
3. From this it follows that gradients must line up (are lin dependent).
4. This then proves the "gradient" statement of the Method
5. The formal Method is the same as saying the gradients line up.

I felt that my proof was "clever" because it was done with the R matrix and determinants, and I put in all kinds of appendix stuff on determinants and rank.

I did not realize that it is very easy to prove this theorem without much work:

1. If df = 0 with constraints, then the gradients must line up.]

I now have this short and simple proof in my first section, amplified by surface discussions. I just reread the proof. It is not quite "short", and it is definitely "geometric". I define stationary point and I then show that the gradients must line up.

SO, I suppose I could repeat my "long proof" as just an alternative. So maybe I could write a short Section 6 showing how you prove the method without appealing to geometry, just to matrices. I will try to write this up tomorrow and see where it leads. Since this is an alternate proof, I can just do my n=6 example and skip the general proof, thus shortening this section.

Status 10.22.16

I have decided to crib my entire old sections on the R matrix and proof of Theorem 1, but that is now going to be Theorem 3. I can pretty much copy everything and just redo equation numbers. I do have to add some text at the start, explaining what I am doing. This will then all be inside the new Section 6.

Appendix B review
B.1 fine and self contained
etf. I read it all and added a new opening paragraph.

This appendix really does have more than reader needs to know, and I comment on that and suggest that it encapsulates Facts that the reader might find useful elsewhere.

Appendix A review
this is also OK, had to update just a few eq nums which refer to the main text.

Appendices A,B really form a pair and should always stick together.

Appendix C review
Done, and changed a few eq nums. It is still relevant to my matrix proof now in Section 6.2.

Appendix D review.

Should I keep this as an Appendix or should I make it be Section 7 ? I argue for appendix because it deals with functionals and that is out of the main line. Also I won't have to change much!

Appendix D review.
opening comments, I did editing here, now OK

<u>Constraints and virtual displacements</u>

...done

Status at 2:15 PM. I am done reviewing App D, made a few changes.

I am now going to copy over the appendices and references (which I also updated).

Status 2:30. I finally have a full copy of the "new" Lagrange doc. It will need lots of checking, but for the first time all the pieces are in there!

Let's go after an overview! DONE.

Updated the entire Maple Visio index doc, DONE.

Pagination

TOC	OK
Overview	OK
Section 1	OK
Section 2	OK
Section 3	OK
Section 4	OK
Section 5	
5.1	OK
5.2	OK
5.3	OK
5.4	OK
5.5	OK
Section 6	
6.1	OK
6.2	OK
Appendix A	OK
Appendix B	OK
Appendix C	OK
Appendix D	OK
References	OK

Total is 116 pages.

Proofing:

TOC OK, had to update the link to the viewer

Overview OK

Section 1

Preliminary Facts about Surfaces OK

Continue Sunday 10.23.16

I am going to pause the proofing cycle to do some other things first.

What are the big changes I am making in this new Lag doc?

The biggest change is that I have **clarified the "proof"** of the Method. In my old doc I was claiming that the main proof was the long and boring matrix method, but even there I had the logic confused. In my "general case" after the sphere examples I sort of had the "gradient method" proof. Again it was hazy. And nothing was really iff type. This I feel was the main flaw in the paper: bad logic, and failure to identify the simple method of proof. Now I present the simple method right at the start using the tangent space concept. I maintain the matrix proof much later in Section 6. I make all the "proofing logic" very much clearer. Again, this is the biggest change.

Basically I just failed to understand "the gradient method". I was lazy perhaps.

Other content changes?

1. The original did not have Appendix C and Appendix D. The latter is a huge addition.

2. My Boltzmann section has been cleaned up and the continuum case is completely new.

Total page count is now 116. The old doc was 64 pages, so it has roughly doubled in size!

What do other sources have to say?

I review a few of these in my old edit log. Buck, Trench, Wiki. Let's take a new look at sources.

Trench:

Does not use phrase "stationary point" but rather "extreme point".

Uses L_{xi} to show a derivative

g_i are the constraint functions, i = 1 to m. The space of **r** is E^n . **r** is called **X**.

A stationary point is called $\mathbf{X}_0$

Constructs in (6) an R matrix only for the constraint functions. His only point here is to make sure that this determinant does not vanish. For me, this would be the lower left square S-1xS-1 square matrix of my R. If that det were 0, I don't see how that affects me.

On page 3 he then makes the gradient "claim". There is no H function.

He then does "1 constraint" with examples, then "quadratic forms" which for me is just a simple example.

Then "2 constraints". He does everything with R matrix pieces. I do a similar thing in (2.9) where I take a square subset of (2.9) and I have a matrix that is square and filled with partials of the constraint functions. I use this to get the λ_i. I do comment that the square matrix must not have zero det, and he points that out as well.

Lots of the phrase "continuously differentiable".

Section 5: This is C = 2 which he calls m = 2, two constraints. Coordinates to do n. He shows the gradient statement. In (23) he shows what I call (2.9), a matrix for the λ_i. Bottom page 12 he shows a square matrix which for me is the red one in my (6.1.3), the leftmost SxS. He sets this det = 0, but that does not mean all the dets would be 0, so that does not mean fall below full rank.

His (24) looks like my (6.2.11) involving a 0's column vector.

OK enough. His presentation I find very confusing and not "general purpose". I think his Theorem 1 is what I now call Theorem 3.

OK, I will keep Trench in my refs list, but I think my pres is very much clearer.

Wiki:

states my Section 2 official method without proof.

The call my H by name L and refer to it as the Lagrangian or Lagrange function.

Claims stationary points of H with no constraints give the solution.

Some good examples with pics, but this is not anything like my paper, no method proof.

Harvard PDF math 21a -- nothing useful
UCB PDF, nada

People mention the method, but nobody is proving it.

Sawyer PDF: this paper does have a "short proof" but it is not very clear to me. It is like my Section 1 proof which I think is much better.

So OK, nothing screams out at me from the web. Most items are statements of the Method and then a few examples.

Now I will resume my proofing.

Proofing:

TOC OK, had to update the link to the viewer

Overview OK

Section 1

Preliminary Facts about Surfaces	OK
The Intersection Constraint Surface A(**r**) = 0	OK
Definitions: The tangent space and the perp space at **r**	OK
Comment: Avoiding gradient confusion	OK
The Stationary Point Problem	OK to (1.20)

This Section is just plain excellent IMHO. It threads the needle exactly and ends with the Main Point in Theorem 1. I think people will appreciate this derivation with its limited math jargon.

Section 2

opening section = excellent	OK

Stationary Points vs. Extremal Points OK

Comments OK

I think chapter 2 gets high marks from today's reviewer! Comment 4 is finally "right".

Section 3

3.1 Example 1: sphere in E^3 . Very long-winded, looking under every rock. It is fine.
3.2 Example 2: a bit less long-winded. Gradually increases complexity. OK
3.3 Example 3: jsut for fun, very short and OK, earlier are special cases. OK
3.4 Example1 Revisited: just a small piece on peak widths, an analogy with Boltz OK
I think this little section is worth keeping, the idea of a peak width on constraint surface.

Section 4

This is just fine, unchanged really from the original. OK

Section 5 (total length is 34 pages!!! A very extended example indeed)

opening text OK
5.1 Statement of The Boltzmann Extremum Problem OK
5.2 Solution of The Boltzmann Extremum Problem
opening section OK
Application of the Method of Lagrange Multipliers OK
The notion of absolute temperature OK
How the derived Lagrange multiplier values β and A are determined. OK
5.3 More details of the solution OK
5.4 Example: N particles in 3 states : a numerical example OK
5.5 Example: The Maxwell-Boltzmann Distribution (section is 13 long pages!)
The System OK
Connection to Sections 5.1 and 5.2 OK
The Maxwell-Boltzmann speed distribution OK
Mean Values OK
Determination of the derived Lagrange Multiplier A OK
Quantum Mechanics and Determination of V OK
Numeric Values for N(v) and g(v) and g(v)/N(v) OK
Summary OK

Abstract for Researchgate:

The Method of Lagrange Multipliers is a way to find stationary points (including extrema) of a function subject to a set of constraints. The Method is derived twice, once using geometry and again using matrices. Five detailed examples are provided. The first three involve maximizing hemisphere height in 3,4 and N dimensions subject to certain simple constraints. The fourth, which requires numerical work, finds the extremal distances between a circle and an ellipse located in a plane. The fifth is an extended example from statistical mechanics: the Boltzmann distribution. The final section discusses the slightly different use of the Method of Lagrange Multipliers in Lagrangian mechanics.

Section 6 (11 pages)

opening material		OK
6.1. The R matrix and its Rank	∇f	OK
6.2. Proof of Theorem 3 ($\Rightarrow$)		OK

I reviewed the new areas here, I did not review the fine details of the bulk which has not changed since the original Lagrange doc. I am worn down for today.

Continue Monday 10.24.16

Quick review of Appendix A: done, made a few tiny changes, actually read my proofs here.

Quick review of Appendix B: done, made a few tiny changes, actually read my proofs here. Browsed this thing, I did it in detail a day or two ago, no sign changes from original doc, so declare OK.

Quick review of Appendix C: Read the whole thing, added a tiny picture at the end.

Appendix D: STOP. Repaired drawing. RESUME:

Constraints and virtual displacements	OK
Assumption that constraints do no virtual work	STOP. Repaired drawing. RESUME:
Comment on internal forces	OK
An Application of Lagrange Multipliers	OK
Summary of the above Lagrange Multiplier Application	OK
Generalized coordinates, generalized forces, and Lagrange's Equations	OK
The Case of C holonomic constraints treated as if they were non-holonomic	OK
Footnote: Carry out the $\delta S = 0$ functional variation of the action .	OK

References: OK, did some edits here.

This completes another lengthy edit pass of the entire document!

I am now going to do a by-section spell check.

first 40 pages through (5.3.2) : done, found one repeated word error, no spelling errors
second 40 pages through (A.2) : no errors found
last third: no errors found . Pretty amazing really, I guess checked from old doc mostly.

Reread Overview and Summary: OK

Scan for thrown EQ nums: bug found at 5.5.12 in seq cheeck, fixed. Did this in a single scan, if the seq is OK I know nothing is thrown wrap to the left edge.

Review pagination:

TOC OK
Overview OK

Section 1 OK
Section 2 OK
Section 3 OK
Section 4 OK
Section 5 OK
Section 6 OK
App A OK
App B OK
App C OK
App D OK
Refs OK

Added title and name in properties. Done. I am ready for a first PDF on this thing!!

Continue Tuesday 10.25.16

Cleaned up App D "index" to show location of the two Lagrange Mult apps in this appendix.

Rereading all of Appendix D. // Done and again did various small checks and edits. This might be the most significant section in my paper, hidden off as an appendix at the end. I doubt one can find this topic more clearly explained than I have done it here. And it correctly belongs in a paper with the title mine has.

Reread select sections of the rest where I know the text is new since the old PDF.

overview OK

Section 1 is excellent! Right to the point I think. You want the opening section to be clear clean and simple.

Center all pictures:

Section 2: fine.
Section 3: intro OK
Section 4: opening OK
Section 5: this is a magnus opus part of this document

It was my whole motivation for doing the document in the first place, so apropos that I gave it much space.

Section 6: fine

I am ready for the first PDF attempt on Alta! Transfer on jump done, first shot in progress. It fails at once. Try 2nd time. Creating pages. Please wait. Done. Bookmarks OK BUT Appendix D is missing even though in the TOC OK. Seen this before, will just retype and do another cycle. Otherwise bookmarks are all correct.

Fix done, now on Alta again with attempt 3. Creating pages. Please wait. Problem is fixed. Bring back to Black for perusal. Looks good, lunch break then stare at it for a while. Thrown eq nums etc. Bad pagination?

- Maybe get rid of labels 24a and 24b if they are never used! DONE
- page 77 tab problem on lines with eq nums. DONE
- (D.24) is thrown, I am not surprised. DONE.
- top page 102 want line on prev page. DONE
- change ver date to 25th DONE

I did a FULL thrown-eq check and FULL seq number check at the same time, problems as noted above.

Now I will fix all these things and put DONE to the right of each one. Confirm in red from new PDF. (24a) is never referred to. But (24b) has lots of refs. OK, I came up with a good compromise fix.

Check all the above fixes:

Image centering: DONE

Time for another PDF cycle. Transfer, start PDF cycle, creating pages, done. Actually did a few cycles to get (1.24) looking right in both places it appears. Done.

I am going to release now at 1:45 PM. Did so to **Xmission**, done.

Since it is now easy to update a doc on **Researchgate**, I will upload the text to there right now.

Uploaded easily, then logged out, went to their search site and found and downloaded my doc as a check. Works fine, all done.

Wrote history just now at the start of this doc. Hope this is all done!!!

Update 10.29.16

As always happens, I found some errata, and they are not just missing periods and I want to get things updated. But I think I should do more proofing just to ferret out any more, now that things have cooled for a few days.

Overview OK
Section 1 I read this last night and did not find anything bad.
Section 2 same
App D same

Well, I can't simply proof this thing, it is too time-consuming, so I will right now just fix what I have found so far and so a cycle with today's date and push it out to both places.

PDF production in progress. It fails first shot. I restart it. Creating pages. This time Word crashes!!! Word is recovering my documents. I restart Word after this, my jump drive file was not changed. This time it works fine, I bring the file back. I then review all the errata one at a time! // But I found another trivial error so doing another cycle, 8:33 is the time on the doc. // Released to Xmission, done. Date in the index file, and the new doc. Now update on Rgate: done, went very smoothly, just said I uploaded a wrong version. They don't have a "typos" item. ALL DONE.