Rigid Body Drawing for Appendix D PhL 10.24.16

I thought this was trivial, but now I see that I really have to solve the whole problem before I can draw the vectors. So I will now solve the problem. This is the problem **with the string**.

Force on m_1 is $\mathbf{F_1} = m_1\mathbf{g} + \mathbf{R_1} + \mathbf{T}$ where $\mathbf{R_1} = \mathbf{N_1} + \mathbf{T_{12}}$. $\mathbf{T}$ = force of the string

Force on m_2 is $\mathbf{F_2} = m_2\mathbf{g} + \mathbf{R_2}$ where $\mathbf{R_2} = \mathbf{N_2} - \mathbf{T_{12}}$.

Both $\mathbf{F_1}$ and $\mathbf{F_2}$ must point along the ramp.

Both masses have the same acceleration so $\mathbf{a_1} = \mathbf{a_2}$ which means $\mathbf{F_1}/m_1 = \mathbf{F_2}/m_2$ or $m_2\mathbf{F_1} = m_1\mathbf{F_2}$.

Aside: Does this give some preliminary contradiction?

$$m_2(m_1\mathbf{g} + \mathbf{R_1} + \mathbf{T}) = m_1(m_2\mathbf{g} + \mathbf{R_2})$$

or

$$m_2(\mathbf{R_1} + \mathbf{T}) = m_1\mathbf{R_2} .$$

Now the two $\mathbf{R_i}$ vectors are not in the same direction. Write this in more detail

$$m_2(\mathbf{N_1} + \mathbf{T_{12}} + \mathbf{T}) = m_1(\mathbf{N_2} - \mathbf{T_{12}})$$

or

$$m_2\mathbf{N_1} - m_1\mathbf{N_2} = -m_2\mathbf{T} - m_2\mathbf{T_{12}} - m_1\mathbf{T_{12}}$$

or

$$m_1\mathbf{N_2} - m_2\mathbf{N_1} = m_2\mathbf{T} + m_2\mathbf{T_{12}} + m_1\mathbf{T_{12}}$$

or

$$m_1\mathbf{N_2} - m_2\mathbf{N_1} = m_2\mathbf{T} + (m_1+m_2)\mathbf{T_{12}}$$

All terms on the left are in the $\hat{\mathbf{y}}'$ direction, while all terms on the right are in the $\hat{\mathbf{x}}'$ direction. Thus each side is separately zero and we then have

$$m_1\mathbf{N_2} = m_2\mathbf{N_1} \qquad \mathbf{N_2} = (m_2/m_1)\mathbf{N_1} \qquad \mathbf{N_2} = \frac{m_2}{m_1}\mathbf{N_1}$$

$$m_2\mathbf{T} = -(m_1+m_2)\mathbf{T_{12\sigma}}$$

Now suppose the string is just holding it back a little bit, a very small T to ghe left. Then the first T_{12} is to the left as well.

$$\mathbf{T_{12}} = -\frac{m_2}{m_1+m_2}\mathbf{T}$$

$$\mathbf{F_1} = m_1\mathbf{g} + \mathbf{N_1} + \mathbf{T_{12}} + \mathbf{T}$$

$$\mathbf{T}_{12} + \mathbf{T} = -\frac{m_2}{m_1+m_2}\mathbf{T} + \mathbf{T} = -\frac{m_2}{m_1+m_2}\mathbf{T} + \frac{m_1+m_2}{m_1+m_2}\mathbf{T}$$

$$= \frac{m_1}{m_1+m_2}\mathbf{T}$$

So

$$\mathbf{F}_1 = m_1\mathbf{g} + \mathbf{N}_1 + \frac{m_1}{m_1+m_2}\mathbf{T}$$

$$\mathbf{F}_2 = m_2\mathbf{g} + \mathbf{N}_2 - \mathbf{T}_{12} = m_2\mathbf{g} + \mathbf{N}_2 + \frac{m_2}{m_1+m_2}\mathbf{T}$$

Summarize:

$$\mathbf{F}_1 = m_1\mathbf{g} + \mathbf{N}_1 + \frac{m_1}{m_1+m_2}\mathbf{T}$$

$$\mathbf{F}_2 = m_2\mathbf{g} + \mathbf{N}_2 + \frac{m_2}{m_1+m_2}\mathbf{T}$$

Accelerations:

$$\mathbf{a}_1 = \mathbf{g} + [\,\mathbf{N}_1 + \frac{m_1}{m_1+m_2}\mathbf{T}\,]/m_1$$

$$\mathbf{a}_2 = \mathbf{g} + [\,\mathbf{N}_2 + \frac{m_2}{m_1+m_2}\mathbf{T}]/m_2$$

$$\hat{\mathbf{z}} \bullet \mathbf{a}_1 = \hat{\mathbf{z}} \bullet g + [\,\frac{m_1}{m_1+m_2}\mathbf{T}\,]/m_1$$

$$\hat{\mathbf{z}} \bullet \mathbf{a}_2 = \hat{\mathbf{z}} \bullet \mathbf{g} + [\,\frac{m_2}{m_1+m_2}\mathbf{T}]/m_2$$

or

$$\hat{\mathbf{z}} \bullet \mathbf{a}_1 = g\sin\theta + \frac{1}{m_1+m_2}T \qquad \mathbf{T} = T\hat{\mathbf{z}}$$

$$\hat{\mathbf{z}} \bullet \mathbf{a}_2 = g\sin\theta + \frac{1}{m_1+m_2}T$$

$$a_{1z} = a_{2z} = g\sin\theta - \frac{1}{m_1+m_2}|\mathbf{T}|$$

Now I know that both these forces are along the ramp. Let $\hat{\mathbf{z}}$ be down the ramp and $\hat{\mathbf{n}}$ be normal to the ramp. Then

$$\hat{\mathbf{n}} \bullet \mathbf{F}_1 = m_1 g\cos\theta + N_1$$

$$\hat{\mathbf{n}} \bullet \mathbf{F}_2 = m_2 g\cos\theta + N_2$$

But we know F_1 and F_2 are down the ramp, so

$$N_1 = -m_1 g \cos\theta$$
$$N_2 = -m_2 g \cos\theta$$

These facts are unaffected by T and the stick and so on. Next,

$$\hat{\mathbf{z}} \bullet \mathbf{F}_1 = m_1 g \sin\theta + \frac{m_1}{m_1+m_2} T$$

$$\hat{\mathbf{z}} \bullet \mathbf{F}_2 = m_2 g \sin\theta + \frac{m_2}{m_1+m_2} T \qquad T < 0 \text{ in my picture}$$

So rewrite as

$$F_1 = m_1 g \sin\theta + \frac{m_1}{m_1+m_2} T$$

$$F_2 = m_2 g \sin\theta + \frac{m_2}{m_1+m_2} T \qquad T < 0 \text{ in my picture}$$

Note that $F_1/m_1 = F_2/m_2$. Now consider

$$\mathbf{R}_1 = \mathbf{N}_1 + \mathbf{T}_{12}$$
$$\mathbf{R}_2 = \mathbf{N}_2 - \mathbf{T}_{12}$$

$$\hat{\mathbf{n}} \bullet \mathbf{R}_1 = N_1$$
$$\hat{\mathbf{n}} \bullet \mathbf{R}_2 = N_2$$

$$\hat{\mathbf{z}} \bullet \mathbf{R}_1 = T_{12}$$
$$\hat{\mathbf{z}} \bullet \mathbf{R}_2 = -T_{12}$$

so these are indeed equal and opposite.