Rigid Body Drawing for Appendix D PhL 10.24.16

I thought this was trivial, but now I see that I really have to solve the whole problem before I can draw the vectors. So I will now solve the problem.

Paradox: I solve the problem below and the result is a contradiction . Watch:

Force on m_1 is $\mathbf{F_1} = m_1\mathbf{g} + \mathbf{R_1}$ where $\mathbf{R_1} = \mathbf{N_1} + \mathbf{T_{12}}$.

Force on m_2 is $\mathbf{F_2} = m_2\mathbf{g} + \mathbf{R_2}$ where $\mathbf{R_2} = \mathbf{N_2} - \mathbf{T_{12}}$.

Both $\mathbf{F_1}$ and $\mathbf{F_2}$ must point along the ramp.

Both masses have the same acceleration so $\mathbf{a_1} = \mathbf{a_2}$ which means $\mathbf{F_1}/m_1 = \mathbf{F_2}/m_2$ or $m_2\mathbf{F_1} = m_1\mathbf{F_2}$.

Aside: Does this give some preliminary contradiction?

$$m_2(m_1\mathbf{g} + \mathbf{R_1}) = m_1(m_2\mathbf{g} + \mathbf{R_2})$$

or

$$m_2\mathbf{R_1} = m_1\mathbf{R_2}$$

This does seem odd, it says that the constraint forces point in the same direction ! Keep going:

$$m_2(\mathbf{N_1} + \mathbf{T_{12}}) = m_1(\mathbf{N_2} - \mathbf{T_{12}})$$

$$m_2\mathbf{N_1} - m_1\mathbf{N_2} = -(m_1+m_2)\mathbf{T_{12}}$$

More contradiction! Since both normal forces are perp to the ramp, this says $\mathbf{T_{12}}$ must be perp to the ramp.

So I can stop right here, no need to do algebra below, already have the Paradox.

So let $\hat{\mathbf{x}}$ and $\hat{\mathbf{y}}$ be the usual axes to the right and up, and let $\hat{\mathbf{x}}'$ and $\hat{\mathbf{y}}'$ be rotated axes so $\hat{\mathbf{x}}'$ is down the ramp.

$m_1\mathbf{g} = -m_1 g\ \hat{\mathbf{y}}$ $\quad\quad$ $\mathbf{F_1} = F_1\hat{\mathbf{x}}'$ $\quad\quad$ $\mathbf{T_{12}} = T_{12}\ \hat{\mathbf{x}}'$

$m_2\mathbf{g} = -m_2 g\ \hat{\mathbf{y}}$ $\quad\quad$ $\mathbf{F_2} = F_2\hat{\mathbf{x}}'$

Then equations become

Force on m_1 is $\mathbf{F_1} = m_1\mathbf{g} + \mathbf{R_1} = -m_1 g\ \hat{\mathbf{y}} + \mathbf{R_1}$ $\quad\quad$ $\mathbf{N_1} = N_1\hat{\mathbf{y}}'$ $\quad\quad$ $N_1 > 0$

$\mathbf{R_1} = \mathbf{N_1} + \mathbf{T_{12}} \quad \Leftrightarrow \quad \mathbf{R_1} = N_1 \hat{\mathbf{y}}' + T_{12}\ \hat{\mathbf{x}}'$

so then

$\mathbf{F_1} = -m_1 g\ \hat{\mathbf{y}} + N_1 \hat{\mathbf{y}}' + T_{12}\ \hat{\mathbf{x}}'$

Force on m_2 is $\mathbf{F_2} = m_2\mathbf{g} + \mathbf{R_2} = -m_2 g\ \hat{\mathbf{y}} + \mathbf{R_2} \qquad \mathbf{N_2} = N_2 \hat{\mathbf{y}}' \qquad N_2 > 0$

$\mathbf{R_2} = \mathbf{N_2} - \mathbf{T_{12}} \quad \Leftrightarrow \quad \mathbf{R_2} = N_2 \hat{\mathbf{y}}' - T_{12}\ \hat{\mathbf{x}}'$

so then

$\mathbf{F_2} = -m_2 g\ \hat{\mathbf{y}} + N_2 \hat{\mathbf{y}}' - T_{12}\ \hat{\mathbf{x}}'$

Now write out $\mathbf{F_1}/m_1 = \mathbf{F_2}/m_2$

$[\, -m_1 g\ \hat{\mathbf{y}} + N_1 \hat{\mathbf{y}}' + T_{12}\ \hat{\mathbf{x}}' \,] / m_1 = [\, -m_2 g\ \hat{\mathbf{y}} + N_2 \hat{\mathbf{y}}' - T_{12}\ \hat{\mathbf{x}}' \,] / m_2$

I now need to write

$\hat{\mathbf{x}}'$ = down slope $= (\cos\theta\ \hat{\mathbf{x}} - \sin\theta\ \hat{\mathbf{y}})$

$\hat{\mathbf{y}}'$ = perp to slope $= (\cos\theta\ \hat{\mathbf{y}} + \sin\theta\ \hat{\mathbf{x}})$

Now the acceleration equation tells us

$[\, -m_1 g\ \hat{\mathbf{y}} + N_1 (\cos\theta\ \hat{\mathbf{y}} + \sin\theta\ \hat{\mathbf{x}}) + T_{12}\ (\cos\theta\ \hat{\mathbf{x}} - \sin\theta\ \hat{\mathbf{y}}) \,] / m_1$

$= [\, -m_2 g\ \hat{\mathbf{y}} + N_2 (\cos\theta\ \hat{\mathbf{y}} + \sin\theta\ \hat{\mathbf{x}}) - T_{12}\ (\cos\theta\ \hat{\mathbf{x}} - \sin\theta\ \hat{\mathbf{y}}) \,] / m_2$

We can balance the x and y components separately to get two equations:
First here is the $\hat{\mathbf{x}}$ equation followed by the y equation

$[\, N_1 (\sin\theta\ \hat{\mathbf{x}}) + T_{12}\ (\cos\theta\ \hat{\mathbf{x}}) \,] / m_1 = [\, N_2 (\sin\theta\ \hat{\mathbf{x}}) - T_{12}\ (\cos\theta\ \hat{\mathbf{x}}) \,] / m_2$

$[\, -m_1 g\ \hat{\mathbf{y}} + N_1 (\cos\theta\ \hat{\mathbf{y}}) + T_{12}\ (-\sin\theta\ \hat{\mathbf{y}}) \,] / m_1 = [\, -m_2 g\ \hat{\mathbf{y}} + N_2 (\cos\theta\ \hat{\mathbf{y}}) - T_{12}\ (-\sin\theta\ \hat{\mathbf{y}}) \,] / m_2$

Rewrite these as

$[\, N_1 \sin\theta + T_{12}\cos\theta \,] / m_1 = [\, N_2 \sin\theta - T_{12}\cos\theta \,] / m_2$

$[\, -m_1 g + N_1 \cos\theta - T_{12}\sin\theta \,] / m_1 = [\, -m_2 g + N_2 \cos\theta + T_{12} \sin\theta \,] / m_2$

Known quantities are m_1, m_2, g, $\sin\theta$, $\cos\theta$. Simplify

$$m_2[\ N_1 \sin\theta + T_{12}\cos\theta\ ] = m_1[\ N_2 \sin\theta - T_{12}\cos\theta\ ]$$

$$m_2[\ -m_1 g + N_1\cos\theta - T_{12}\sin\theta\ ] = m_1[\ -m_2 g + N_2\cos\theta + T_{12}\sin\theta\ ]$$

$$[\ N_1 m_2\sin\theta + m_2 T_{12}\cos\theta\ ] = [\ N_2 m_1\sin\theta - m_1 T_{12}\cos\theta\ ]$$

$$[\ -m_1 m_2 g + N_1 m_2\cos\theta - T_{12} m_2\sin\theta\ ] = [\ -m_1 m_2 g + N_2 m_1\cos\theta + T_{12} m_1\sin\theta\ ]$$

$$[\ m_2\sin\theta N_1 + m_2\cos\theta T_{12}\ ] = [\ m_1\sin\theta N_2 - m_1\cos\theta T_{12}\ ]$$

$$[\ m_2\cos\theta N_1 - m_2\sin\theta T_{12}\ ] = [\ m_1\cos\theta N_2 + m_1\sin\theta T_{12}\ ]$$

$$m_2\sin\theta N_1 - m_1\sin\theta N_2 = -m_2\cos\theta T_{12} - m_1\cos\theta T_{12}$$

$$m_2\cos\theta N_1 - m_1\cos\theta N_2 = m_1\sin\theta T_{12} + m_2\sin\theta T_{12}$$

$$(m_2 N_1 - m_1 N_2)\sin\theta = (-m_2 - m_1) T_{12}\cos\theta$$

$$(m_2 N_1 - m_1 N_2)\cos\theta = (m_1 + m_2) T_{12}\sin\theta$$

$$(m_2 N_1 - m_1 N_2) = -(m_2 + m_1) T_{12}\cot\theta$$

$$(m_2 N_1 - m_1 N_2) = (m_1 + m_2) T_{12}\tan\theta$$

Something is wrong because the two left sides are the same and the right sides are not the same. To make the right sides the same you would need to have

$$-(m_2 + m_1) T_{12}\cot\theta = (m_1 + m_2) T_{12}\tan\theta$$

or

$$-\cot\theta = \tan\theta$$

and this is a contradiction, confirming the Paradox.