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Overview and Summary

The intent of the presentation below is to provide a *reasonable* derivation of why the Method of Lagrange Multipliers works. Although we do discuss the relevant "gradient approach", it seems perhaps more convincing to use a "matrix approach" where the solution points of a constrained extremum problem are associated with the points at which a certain matrix drops below full rank. Then the gradient statement of "why it works" follows at once. It certainly is a mind-boggling exercise to attempt to visualize the solution sets as intersections of multiple surfaces in a space of more than three dimensions.

Every document on this subject needs a "Theorem 1" and ours is no exception. Fortunately for the reader, there are no other theorems, and Theorem 1 is fairly easy to prove. The only required reader investment concerns basic linear algebra facts, in particular, the notion of the rank of a non-square matrix.

We first state our Theorem 1 in Section 1, then show in Section 2 how it provides an immediate derivation of the Method of Lagrange Multipliers. Section 3 then proves the theorem. Section 4 gives the gradient interpretation and provides an example which at least can be visualized with simple surfaces. Section 5 does another example which, though simple, requires numeric work to solve. Finally, Section 6 provides an extended standard example involving statistical mechanics.

What the reader will not find below is a treatment of questions of continuity assumptions and whether solutions do or do not exist for special situations. This subject seems well treated in the literature.

The general tone is more that of an engineer or physicist, not that of an abstract mathematician. Maple code is used where it seems useful to implement calculations or display graphs.

When an earlier equation is quoted, its equation number is put in italics.

A short list of references is provided on the last page.