Edit of Comment 4.

this is obs

4. The function H in (2.1) is sometimes referred to as **the Lagrangian** and is written L. This Lagrangian differs from that which appears in the Lagrangian formulation of particle motion which we describe in Appendix D. Here we show the connection.

In (2.1) the function H is defined by,

$$H(\mathbf{r},\boldsymbol{\lambda}) \equiv f(\mathbf{r}) + \lambda_1 a(\mathbf{r}) + \lambda_2\, b(\mathbf{r}) + \ldots\ldots + \lambda_{s-1}\, q(\mathbf{r}) \ . \qquad \textit{(2.1)}$$

We must now rename the constraint functions using one of the methods from Comment 2 above, and this means for the moment that we give up the notation that $a_i \equiv \partial a/\partial x_i$. Setting S-1 = C and defining $a_1 = a$, $a_2 = b$. .... , $a_C = q$, the above equation for H becomes

$$H(\mathbf{r},\boldsymbol{\lambda}) \equiv f(\mathbf{r}) + \Sigma_{i=1}^{C} \lambda_i a_i(\mathbf{r}) \ . \qquad (2.10)$$

We require that

$$dH = 0 \qquad (2.11)$$

from which we conclude in (2.4) that $a_i(\mathbf{r}) = 0$ and

$$\frac{\partial H}{\partial x_k} = \frac{\partial f(\mathbf{r})}{\partial x_k} + \Sigma_{i=1}^{C} \lambda_i \frac{\partial a_i(\mathbf{r})}{\partial x_k} = 0 \qquad k = 1,2,....N \ . \qquad (2.12)$$

The N coordinates x_k are the components of $\mathbf{r} = (x_1, x_2, ...x_N)$.

In Appendix D the object that most closely corresponds to H is the action S, where

$$S = \int_{t1}^{t2} dt\, L(q_n(t), \dot{q}_n(t), t) \ . \qquad (2.13)$$

The N coordinates q_k are the generalized coordinates of the Lagrange problem.

It is OK to refer to the function H in (2.10) as "the Lagrangian", but H really corresponds to S which is in fact a certain time integral of the classical mechanics Lagrangian L. In the presence of C holonomic constraints $a_i(q_i,t) = 0$, we require, similar to (2.11), that

$$\delta S = 0 \ . \qquad (2.14)$$

This produces a set of modified Euler-Lagrange equations (D.54) which closely resemble the set (2.12),

$$\left(\frac{\partial L}{\partial q_k} - \frac{d}{dt}\frac{\partial L}{\partial \dot{q}_k}\right) + \Sigma_{i=1}^{c} \lambda_i \frac{\partial a_i}{\partial q_k} = 0 \qquad k = 1,2...N \qquad (2.15)$$

Here the role played by $\frac{\partial f}{\partial x_k}$ in (2.12) is assumed by $\left(\frac{\partial L}{\partial q_k} - \frac{d}{dt}\frac{\partial L}{\partial \dot{q}_k}\right)$ in (2.15) .