Edit of Comment 4.

4.The function H in (2.1) is sometimes referred to as **the Lagrangian** and is written as L. This Lagrangian differs from that which appears in the Lagrangian formulation of particle motion which we describe in Appendix D. Here we show the connection.

In (2.1) the function H is defined by,

$$H(\mathbf{r},\boldsymbol{\lambda}) \equiv f(\mathbf{r}) + \lambda_1 a(\mathbf{r}) + \lambda_2\, b(\mathbf{r}) + \ldots\ldots + \lambda_{S-1}\, q(\mathbf{r}) \ . \qquad \textit{(2.1)}$$

Setting S-1 = C and defining $a = a^{(1)}$, $b = a^{(2)}$ and so on, this becomes

$$H(\mathbf{r},\boldsymbol{\lambda}) \equiv f(\mathbf{r}) + \Sigma_{i=1}^{C} \lambda_i a^{(i)}(\mathbf{r}) \ . \qquad (2.10)$$

We require that

$$dH = 0 \qquad (2.11)$$

from which we conclude in (2.4) that $a^{(i)}(\mathbf{r}) = 0$ and

$$H_k = f_k + \Sigma_{i=1}^{C} \lambda_i a_k^{(i)}(\mathbf{r}) = 0 \ . \qquad k = 1,2...N \qquad (2.12)$$

The N coordinates x_k are the components of $\mathbf{r} = (x_1, x_2, \ldots x_N)$.

In Appendix D the object that most closely corresponds to H is the action S, where

$$S = \int_{t1}^{t2} dt\, L(q_n(t), \dot{q}_n(t), t) \ . \qquad (2.13)$$

The N coordinates q_k are the generalized coordinates of the Lagrange problem.

It is OK to refer to the function H in (2.10) as "the Lagrangian", but H really corresponds to S which is in fact a certain time integral of the classical mechanics Lagrangian L. In the presence of C non-holonomic constraints, we require, similar to (2.11), that

$$\delta S = 0 \ . \qquad (2.14)$$

This produces a set of modified Euler-Lagrange equations (D.53) which vaguely resembles (2.12),

$$\left(\frac{\partial L}{\partial q_k} - \frac{d}{dt}\frac{\partial L}{\partial \dot{q}_k}\right) + \Sigma_{i=1}^{C} \lambda_i A_{ik} = 0 \qquad k = 1,2...N \qquad (2.15)$$

Here the role played by f_k in (2.12) is assumed by $\left(\frac{\partial L}{\partial q_k} - \frac{d}{dt}\frac{\partial L}{\partial \dot{q}_k}\right)$ in (2.15), while the role played by the holonomic constraint gradients $a_k^{(i)}$ is assumed by the non-holonomic constraint functions A_{ik} .