Lagrange Multipliers Edit Log PhL 6.1.15

I am searching right now for Linear Algebra standard books so I can quote theorems from somewhere! I have a few obscure texts. I downloaded Shaum, but it does not have the subdeterminant theorem I need. I see that Shilov is perhaps a good standard, now in Dover, so I am getting it right now. Got it finally, it was slow. The Dover is 1977, perhaps the book is 1971. The doc is not searchable, so I will fix that. Book is 399 pages. Xchange can do OCR now, but I don't know how good it is, it is taking about 2 seconds per page so will be 800 sec = 16 minutes. Rank does appear in the index.

I did OCR and Shilov does have exactly what I need. So how I have a very small section on rank which has no more than what is necessary (no need for a Matrix Appendix since I have references to a commonly available source!).

I then did a edit of the first two sections with this newly installed material.

Question: Why did I restrict to $S \leq N$?? I attempted an answer!

Entry June 6, 2015. About a week has passed, and I now have what I think is a reasonable document of about 40 pages. It is finally "stable" and I have answered all "questions" that came up in the process.

I would like now to peruse some other sources just to see if there is anything major I have missed.

1. Buck. This source is what put me onto the matrix rank method in the first place. I think I am familiar with their short section. They only have a tiny section, but it contained the seed of my whole approach.

2. Trench's PDF. This is a 31 page effort. This PDF is a 31 page supplement to his 600 page book, which I just downloaded just to have it. Trench just dives right into the method without saying much about why it works. He uses g_i for the constraint functions. He does "one constraint", and then suddenly wanders off into quadratic forms, but this is just another one-constraint same problem. Why does he deal with the λ_i as eigenvalues here?? Well of the f you want to maximize is quadratic in the variables, then the H_i equation will be linear and you quickly get an $A\mathbf{x} = \lambda\mathbf{x}$ structure and you need to have $\det(A-\lambda) = 0$ to have a non-zero solution, so that is why the single Lagrange λ must be an eigenvalue. Fine. It is a very special case but an interesting one. He then moves into his 2 constraints rework of everything. Here he has the R matrix without the f row and he solves for the λ_i which I have already included. He then shows the full R matrix having a zero determinant. I think this is just one of my SxS = 3x3 minors.

He also has a section Proof of Theorem 1. I just don't really grok what he is doing! He does have lots of small examples. Somehow his method just leaves me cold, though I surely could parse it if I wanted.

3. Wiki page on LM. This is about 15 pages long. They mention the gradient which is why I added my section on that topic. I think I have the gradient under control. There really is not too much of interest to me on this web page, it is not bad

4. Other web. There are lots of offerings. Lectures are usually brief for a course, heavy on examples.

I am now ready to do some proofing. I wrote a simple overview and summary and may change ig.

Overview
text OK

Section 1
text OK
eq num sequence OK

STOP. A perused Shilov a bit, and amazingly concocted my own proof for rank = column rank = row rank. Now start proofing over again.

Overview
text OK, Buck credited now

Section 1
text very good,
eq num all OK

Section 2
text very good
eq num OK

Section 3 continue here on 6/8/15.
text very good OK
eq num OK

Section 4
text very good, generalization is convincing I think
eq num OK

Section 5
not very exciting, but a good numerical example
eq nums OK

Section 6
6.1 statement of the problem eq num OK
6.2 solution of problem good eq num OK
6.3 further math, all good eq nums OK
6.4 numerical with plots all good eq nums OK

Appendix A
text done
eq num OK

References
text OK

All done with proofing.

Pagination. Decided to have each "chapter" start on new page to give me flexibility when I have to edit this document which will surely happen. So then pagination is by Chapter

TOC	ok
Overview	ok
Section 1	ok thru p 5
Section 2	ok thru p 8
Section 3	ok thru p 16
Section 4	ok thru p 21
Section 5	ok thru p 28, not great
Section 6	ok thru p 42
Appendix A	ok thru 45
Refs	ok, half page

All done with pagination. Next come the indices. Well, I tried a PDF on Alta. First I digressed trying once again to get Acro 6 and 8 running on Black, but you can't print even out of Acro 6 and then 8 has an install error, this time I took notes!

So how did the first cut PDF come out? OK, I fixed a few things on Black not on Alta. bookmarks all OK.

I repasted lots of Maple stuff to make it look better, did a final eq num check, and did a PDF so this may be ready for release. I will read the PDF.

- below (1.4), swap rows and columns!!!!!

- above (1.5), say intersection surface instead of intersecting surface

- below (1.9. replace Section 1.3 with Section 3

- add period to (2.4)

- below (3.14) perhaps say "with $i \neq j \neq k$ "

- below (3.2)' say "theoretically"

- below (4.4) should have ∇a in place of ∇f !

- below (4.8) should be one, not on

- In section 4 I need somewhere to say that whatever a surface is, the gradient is the **normal** to it. I sort of skip over this fact. I assume it, but the reader may not know that fact.

I will continue this PDF proofing pass tomorrow, I guess I did not make it today!

- Above 5.1 "our Example 2"
- (5.14) needs a period
- (6.1.2) fix spacing near =
- (6.2.2) clean up spacing
- (6.2.16) clean up spacing

Pause near (6.2.16). consider:

$$Z \equiv \Sigma_i g_i e^{-\beta \varepsilon_i}$$

$$\partial_\beta Z = - \Sigma_i g_i \varepsilon_i e^{-\beta \varepsilon_i}$$

Then we really have

$$- \partial_\beta Z = u Z \qquad \Rightarrow \qquad Z = e^{-u\beta} + C$$

Does this help in solving for β ? $\Sigma_i g_i e^{-\beta \varepsilon_i} = e^{-u\beta} + C$? Not really.

$$\Sigma_i [\varepsilon_i - u] g_i \ e^{-\beta \varepsilon_i} = 0$$

Take ln of both sides?

$$\Sigma_i \ln \{ [\varepsilon_i - u] g_i \ e^{-\beta \varepsilon_i} \} = 1$$

$$\Sigma_i \{ \ln \{ [\varepsilon_i - u] g_i \} - \beta \varepsilon_i) = 1$$

$$\Sigma_i \{ C_i - \beta \varepsilon_i) = 1$$

$$\Sigma_i C_i = \beta \Sigma_i \varepsilon_i \qquad \beta = \Sigma_i C_i / \Sigma_i \varepsilon_i$$

Wow! No need to do numerical! I need to fix up a lot of things !

WRONG!!!! What you really have above is

$$\ln \{ \Sigma_i [\varepsilon_i - u] g_i \ e^{-\beta \varepsilon_i} \} = 1$$

and then you are faced with a ln of a sum which is nothing nice.

- Major repairs for solving for β !!!! (not needed, see WRONG above!)
- spacing in 6.3.4
- Below 6.3.5, clarify "Applying...", it is hazy
- 6.3.14 can do better estimate without the 3. (left as was)
- repair 6.4.16 it has a vertical bar!
- above A.2, contrapositive seems the wrong way to say it
- below A.4 "reversing this logic"
- add mentions of (A.2) where apropos!

OK, I certainly found a lot to fix on this PDF edit pass, including some serious stuff. Let's start by making all the cosmetic repairs and red them as they are done. OK, I think App A needs some rewriting, will get to that when rest is done.

Now let's do all the β repair work.

Start at (6.2.16). Oops, I was wrong, no β repair work to do!

Rewrite of Appendix A.

June 9, 2015. I first wrote Appendix B which derives a bunch of determinant stuff from scratch. I think this is a pretty good appendix IMHO, one you won't find elsewhere in this direct form. Now comes the rewrite of Appendix A.

June 10, 2015. The new appendix A and an appendix B were installed at 8 PM today. Thinking of adding my "clever" det(AB) proof at the end of Appendix B.

June 11, 2015. Added det(AB) and a few other things, and I am now happy with Appendices A and B. Worked in name issues with adjoint and minor. Getting close to PDF time again!

I have now proofed App A and B several times. Next they need to be paginated:

Paginate App A:	done
Paginate App B:	done

I am now ready for a new PDF!

STOP. I wanted to add some clarity to the gradient discussion. As I ponder this with Example 1, my whole understanding of the gradient stuff has completely crashed to the ground, ouch! I don't know how to fit Example 1 into the general picture.

June 12, 2015. I have rewritten the gradient section completely and it now seems pretty good. Before installing, I would like once again to look at some external sources since I will now be more simpatico with their various pictures.

wiki: I am much happier now with all this stuff. They don't prove anything.
looked also at a few others. All is OK.

So let's install this new Section 4 after saving off the old section 4. // Done, we are now at 63 pages.

I have only now to proof the new Section 4, review the Overview again, then paginate Section 4. That is it!

Proofing of new Section 4

4.1 OK and eq num OK
4.2 OK and eq num OK
4.3 OK and eq num OK
4.4 OK and eq num OK

Done! And I keep improving the Overview.

Need to repair some blue text in Section 1. It is OK.

Time to paginate Section 4. Done.

Making a June 12 PDF right now. I will proof manana and collect some errata.

Errata: Need to update my Visio Maple index and clean up the folder again.

June 13, 2015. I did various things today which got lost in this edit log in a system crash.

- added section generalizing Example 2 for general f,a,b functions
- added section showing 1D lot with examples of stationary points etc
- started into a general proofing. Here is the proofing history

TOC	ok
Overview	ok
Section 1	ok
Section 2	ok
Section 3	ok
Section 4	just starting

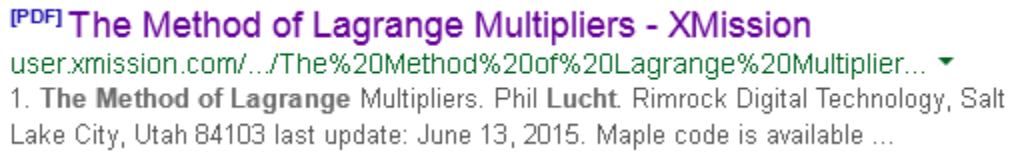
Since I have already gone over Section 4 about 5 times, I will stop this proofing pass. The other sections also have been well proofed. So let's go for a PDF right now at 11:07 AM.

2PM. Putting it on the xmission site right now. Done!! Did a push on both Bing and Google.

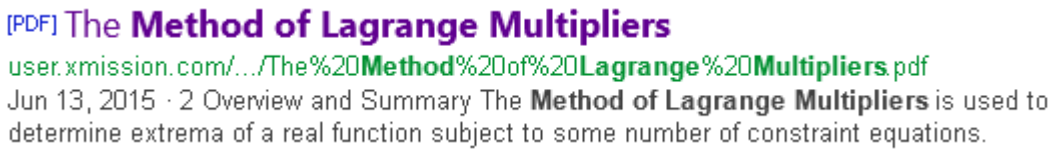
Time investment in Lagrange doc:

Started on this May 17, 2015 after finishing Buck Ch 6 where this subject is mentioned. Did actual work on these days from log: May 17, 18 then pause for Kent/Mem day. Then May 27, 28, 29. Here I am doing "matrix theorems" because I did not know about column rank = row rank = rank. May 30, 31. June 1 is first cut on gradient section. Jun 2, 3, 5, 6 Boltzmann. Doc seems ready at 40 pages. June 7 did Appendix A on the rank theorem. June 8 proofing. June 9 is App B on determinants. June 10 both Apps are installed. June 11, June 12 rewrite gradient section. June 13 some cleanups and proofing and it is out on the web. So basically **16 days** to write a very detailed 64 page document on a specialty subject that has always bugged me.

Update June 15, 2015. I decided to upload to RG after updating bipolar doc there successfully. I opted not to have a DOI created since I see no benefit in doing that. Did a download test this new RG lagrange doc, all is OK, they add that cover page as usual. Amazingly, this Lagrange doc already appears in google search, after a mere 2 days on xmission site. They did the push quickly.



Searching on the title and "pdf" I am on the fifth page of hits. I am on the 4th page of Bing with just the title phrase. Here is the Bing hit.



OK, it is out there now!

Update June 22, 2015

I proofed the entire document today (not check eq nums) and was amazed to find at least 6 moderate typo problems and a total of 12 items in all. I did a cycle and then put out a new Xmission PDF of today's date.

I will now see what RG says. I did NOT make a doi for this doc, in hopes that they allow replacement. But things have changed. How do you now remove text from RG?

Here is my RG abstract just to not lose it

ABSTRACT: The Method of Lagrange Multipliers is derived using a simple matrix method and the result is then interpreted in terms of gradients. Several non-trivial examples are provided including one concerning the Boltzmann distribution. Appendices derive from scratch the necessary determinant theorems for the matrix approach.

There is no longer a way to delete just text, but there is a big REMOVE button so I removed the whole thing, no problem (since no DOI I bet). Then I uploaded a brand new copy and installed the above abstract, no problem at all! This did cause the "stat" of one download of this doc to be deleted. I think RG is in transition on how to delete just the uploaded text, I will come back later and see how things ended up.

Update Oct 2, 2016.

I became concerned about two things. (1) How do you know you can eliminate variables as I do in a key part of my proof of the method. I try to explain this in a new Appendix C. (2) I realized that the classical mechanics usage of the "method of Lagrange multipliers" is considerably different from the method that my paper treats. So I wrote Appendix D to fill in this gap. That took quite a long time to write, and I did it while reviewing Ch 3 of Goldstein. I am now in the process of doing a major update of this date.

* I added the two appendices at the end, and updated the TOC and the overview so these new appendices are mentioned.

* I added the Ince and R&S references to my alpha refs list.

* Added a ref to App C at a point in my proof where this comes up.

* I will now go through ALL the errata and install them. // Partially done 3 PM 10.2, break time. Resumed, and did all I could. I need to proof things again before doing the residual errata.

Appendix B:
- opening OK
- B.1 all OK
- B.2 all OK and long!
- B.3 OK and painful.
- B.4 OK, added lots of periods after equations.

I made no non-cosmetic changes to App B, I am just proofing it for this update.

Question: Is Theorem 11 in (A.4) supposed to apply to a non-square matrix?

Appendix A: OK. Made tiny changes.

Continue Oct 3, 2016.

Added stuff to the end of App D which is then used in edited Comment 4 to make that comment work better, things are now more parallel.

Added a *tail end* to Section B.2 showing a generalized form of my det(M) forms, as suggested in "ways to write" document. Worked in my final "way" result as a reader exercise. No other changes in App B.

All errata are finally made red, so I am ready to start proofing again.

Appendix B.2 tail only : DONE.

Section 1: all OK including no thrown eq nums.

Section 2: all OK including eq num and Comment 4.

Appendix C: all OK inclu eq num

Appendix D:

opening OK

Constraints and virtual displacements	(D.1)	OK
Assumption that constraints do no virtual work	(D.11)	OK
Comment on internal forces	(D.13)	OK
An Application of Lagrange Multipliers	(D.13)	OK
Summary of the above Lagrange Multiplier Application	(D.25)	OK
Generalized coordinates, generalized forces, and Lagrange's Equations	(D.27)	OK
Footnote: Derivation of the result (D.31) used above	(D.40)	OK
Lagrange Equations with non-holonomic constraints	(D.49)	OK
The Case of C holonomic constraints treated as if they were non-holonomic	(D.55)	OK
Footnote: Carry out the $\delta S = 0$ functional variation of the action	(D.56)	OK

Found a mistake below (D.53) which needs a rescue. Fixed. Adding

Added new (D.54) so update entire doc for all following eq nums. 54 and higher all go up by 1.

Continue Oct 4, 2016.

Early AM completed Appendix D proof, including full eq num check and right adjusts. This is I think a great addition to this Lagrange Multipliers document.

So far then I have proofed: A,B,C,D,1,2 . I don't think anything has changed in other sections, but I will proof them all anyway today.

Section 3: Proof of Theorem 1

reviewed Sections 1 and 2 quickly, all OK

(a) Proof for N = 6 and S = 4, OK and very good I think. Tie-in to C is made.

(b) OK, not too bad, all done inclu eq num chk

Section 4. The Gradient Interpretation: Examples 1 and 2

4.1 OK and very good, did make tiny changes.

4.2 OK and Fig (4.2.17) and nearby text has been repaired.

4.3 OK, and made some good repairs.

4.4 OK, read this first. I like my examples, thank you.

gobal eq num chk: OK

What is potentially misleading in this example (so far) is the notion of surface and gradient. A sphere of radius r is described by R(**r**) = r where R(**r**) = $\sqrt{x^2+y^2+z^2}$. In spherical coordinates one easily computes that $\nabla R = (\partial_r R)\,\hat{\mathbf{r}} = \hat{\mathbf{r}}$, and this is indeed normal to the surface at every point on it, as predicted by Fact 2 above. Similarly, the y=1 plane constraint function a(x,y) = y-1 = 0 has $\nabla a = \hat{\mathbf{y}}$ and this is everywhere normal to the constraint plane.

Ouch! My Figure (4.2.16) is screwed up in a few ways. I had to repair things here because the level curve topo lines are for a bowl, not something peaked in the center. Fixed.

Section 5. Example 3: Extremal distances between an ellipse and a circle
A single section, OK, I added a few extra phrases here and there for clarity.

Section 6. Example 4: The Boltzmann factor in Statistical Mechanics
6.1 OK, made some fixes and adds.
6.2 OK, I solve the problem for **r** and the λ_i , **r** is here **N**.
6.3 OK, lots of math here: show max, show width
6.4 OK, checked a few numbers, seems OK with super large numbers.

Almost done.

Overview and Summary: OK, small edits.

References: OK, verified links.

Check all Goldstein page references: all OK

Pagination:
TOC = all on one page, fine
Overview: OK, I spaced out the summary more.
1. OK and very good
2. OK
3. OK
4. OK
5. OK
6. OK
A. OK
B. OK
C. OK, took a little work.
D. OK , this required a full effort.

Not surprisingly, only the new sections required new pagination work. It is done.

Ready for a final glance and then some PDF production. Will do that manana.

Continue Oct 5, 2016.

I rewrote the Overview and think it is now much better, and properly includes Appendix D at a high level. Now doing another quick scan.

Overview OK
Section 1 OK
Section 2 **STOP**. I found a very major logic error and have to do some rewriting! I am no going to copy the existing v1 into v2.

My entire document has just completely collapsed!!!! I have to withdraw it from my website and from Researchgate. It is just plain wrong. Glad I found this and not someone else.

I guess I will start from scratch and write a new document with the same title. Ouch! I should be able to reuse lots of the material, but the logic flow is completely different. Matrices are not needed at all! My appendices A, B, C are then unnecessary, and in fact 90% of my doc is unnecessary.

I have started a bit and it looks good.

Pull Lagrange doc from xmission and Researchgate right now!

xmission: I commented out the line in index.html for this file, and I deleted the file in docs
Researchgate: done, pulled the text and made the abstract one sentence.