Mean Value Theorems PhL 10.14.04

I encountered these while reading up in my Schaum book about Gaussian Quadrature. I was able to find simple proofs of all items in just a few web pages.

Mr Gen-Bin Huang wrote a 28 page paper called "Topics in Mean Value Theorems" where he studies issues relating to the actual values of the ξ type values, an advanced topic for me. But in his first 6 pages he makes a nice statement of four forms of the Mean Value Theorem, and then shows they are pairwise equivalent. I have kept only these pages of his paper. Here are his four forms of the MVT and his names and numbers:

1.1 the **MVT for integrals**. This states that the mean value μ of an integral lies in the range of the function that you are integrating.

1.2 the **generalized MVT for integrals**. This is similar to 1.1, but there is a positive definite weighting function inside the integral, and the (b-a) of 1.1 is replaced by an integral of the weight function. In other words, 1.1 is a special case of 1.2 where the weight function is unity.

1.3 the **classical MVT**. This is the situation shown in the picture of sheet B below. There is some point on a curve segment where the slope matches the overall rise over run. This theorem is proven on sheet C using Rolle's Theorem.

1.4 **Cauchy's MVT**. This one involves two arbitrary functions f and g with the restriction that $g' \neq 0$ over the domain. I don't have a graphic interpretation of this Cauchy MVT which is also called the Extended MVT. Again, if you set $g(x) = x$, you find that 1.3 is a special case of 1.4. This theorem is proven on Sheet A.

Now, Mr Huang shows quickly that (1.1) $\Leftrightarrow$ (1.3) and also that (1.2) $\Leftrightarrow$ (1.4). Since we have proofs of 1.3 and 1.4 on sheets C and A, we then have proofs of all four MVT's.

However, the proof of 1.3 on sheet C requires Rolle's Theorem, so we provide on sheet D a proof of Rolle. All these proofs are quite simple.

A Proof of the Cauchy (extended) MVT

B Graphic picture showing the interpretation of the Classical MVT

C proof of the Classical MVT from Rolle

D proof of Rolle's Theorem