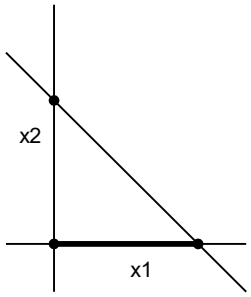
Simplex Example with n=1 and m=1 PhL 11.26.04

This subject appears in Scheid chapter 27. I got sufficiently confused that I am breaking out this separate document for more details.

I need to understand the general idea in a very simple case.

1. Simplest case I can think of. n = 1 and m = 1

Let n =1 and our volume is a segment of the x_1 axis between 0 and 5. We have m = 1 conditions in addition to $x_1 \geq 0$ which is this: $x_1 \leq 5$. We introduce a slack variable x_2 and say $x_1 + x_2 = 5$. This is a line in 2D space. The line is our "boundary" in E^2 , the extended space of E^1 to include the slack variable.



[In the extended space E^2 where n+m = 1+1=2, the feasible region is now the above triangle.]

We want to start off at a "vertex" of our space. Our choices are $x_1 = 0$ or $x_1 = 5$. Let's go with the latter. Then we have

$\mathbf{x} = (5,0)$ // position vector in E^2 to start off with, [also on original region]

Now, we want to move in a [Scheid's] k direction, which here can only be the x_2 direction so k = 2. If we want to stay on the boundary. We want to go distance p in the x_2 direction, so we are going to have

$\mathbf{x'} = (5 - \delta, p)$ // δ to be found below

This must still be on the line, so we know that $(5-\delta) + p = 5$, so $\delta = p$. Thus, our point is

$\mathbf{x'} = (5 - p, p) = (x_1', x_2')$

What is our matrix form of the boundary conditions?

$$[\,1 \quad 1\,] \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = 5 = [\, v_{11} \;\; v_{12}\,] \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = [\, \mathbf{v_1} \;\; \mathbf{v_1}\,] \begin{pmatrix} x_1 \\ x_2 \end{pmatrix}$$

We have m=1 independent column vectors and it is just $v_1 = 1$. Our $H = x_1 c_1$, that is all here is.

[OK to here]

So, we now want to "head off" in the x_2 direction but we want to stay on the boundary [in E^2], so we wander up the line shown. Eventually we will hit a point where $x_1' = 0$, and that is the top dot. We know that $p = 5$ will take us to that point. At that dot, we know that $H = x_1 c_1 = 0$. If we are doing a max problem, this is surely smaller than the point we started with.

Now look at some other Scheid equations:

(3) $\mathbf{v_j} = v_{1j}\,\mathbf{v_1}$ $j = 1,2$ which really says in our case $\mathbf{v_1} = \mathbf{v_1}$ and $\mathbf{v_2} = \mathbf{v_1}$

(4) $h_j = v_{1j}\,c_1 - c_j$ $j = 1,2$ $h_1 = 0$ $h_2 = c_1 - c_2$

I now realize that this Scheid equation is "illogical" because there is no c_2 in the problem, yet he writes it here. For the moment, I will pretend c_2 exists and write this out as shown to the right above:

(5) If $p = 0$, his equation says: $x_1\,\mathbf{v_1} = \mathbf{5}$ or $x_1 = 5$

If $p > 0$ we get instead: $(x_1 - pv_{12})\,\mathbf{v_1} + p\,\mathbf{v_2} = \mathbf{5}$ or $(x_1 - p) + p = 5$

What is the meaning of this equation? I used to thing we were going off in the x_2 direction by amount p, but that is completely wrong. So my entire Scheid has now collapsed into nothingness!

(6) $(x_1 - pv_{12})\,c + pc_2 = x_1 c_1 - p\,[\,c_1 - c_2\,]$

[See official Scheid notes on this stuff.]