Matrix Binder Contents: 12.22.04

This binder contains basic definitions and theorems concerning finite matrices whose elements are real or complex numbers. Almost all theorems concern square matrices. Advanced topics like Jordan Canonical Form are stored in a separate binder. Think of the contents of this binder as being "frozen".

Index of Matrix Theorems. This 13 page document simply lists off all the "theorems" that are contained in the various documents in this binder. Many theorems are extremely basic. All theorems are proven in the supporting documents.

Margenau & Murphy Chapter 10 on Matrices. These are notes I wrote while reading that 32-page chapter, done section by section. M&M is a very old book (2nd Ed 1956), but it was the only book I owned that has a chapter on this subject. Here is the structure of this chapter and of my notes:

10.2 Determinants
10.3 Minors and Cofactors
10.4 det(AB) = det(A)det(B) and rule that ∂ det(A)/∂ a_{pq} = cof(a_{pq}).
10.5 rank and meaning of singular
10.6 combining matrices: conformable, dot product, direct product, commutators
10.7 special matrices: their table showing all the famous names
10.8 vector spaces, Gram Determinant and G-S orthogonalization
10.9 linear equations, linear independence of rows and columns, etc. Nullspace, Cramer's Rule
10.10 linear transformations
10.11 Equivalent matrices, and special cases thereof (similar, congruent, etc)
10.12 Bilinear and quadratic forms, relation to normal modes
10.13 Similarity Transformations
10.14 The characteristic equation of a matrix
10.15 Reduction to diagonal form (a brutal section for me), Pivot and Twist
10.16 Congruence Transformations
10.17 Orthogonal Transformations
10.18,19,20,21 Hermitian Vector Space, Hermitian Matrices, Unitary matrices, wrap up

Gram-Schmidt Orthogonalization Notes. I show that you are doing a basis change of the form U = VR where R is an upper triangular matrix, and how this is an example of the QR factorization.

Some Matrix Theorems. This was written while I read the M&M chapter 10, pieces also from Stakgold. Topics are algebraic versus geometric multiplicity, no defects in Hermitian H and eigenmanifolds of H partition V, eigenvalues of H are real, can diag by similarity, basic theorems about rank and nullity.

The Block Diagonal Form Theorem. I was struggling to show that you can bring *any* matrix to block diagonal form by a similarity. M&M give part of the story with their Pivot and Twist argument, but were unconvincing. I tried here to fill in the missing details. I *now* know that you can similarize any matrix into a block diagonal form because its minimal polynomial matches the form used in the Primary Decomposition Theorem which lets you similarise into a direct sum of the eigenmanifolds. Moreover all blocks can be made triangular in the same direction. Further, you can break each eigenmanifold into smaller blocks to get the final Jordan form, and all this can be done by a similarity. They mention none of this "fancy" stuff. The Schur Decomposition just tells you that you can arrive at an overall triangular form by a similarity.

M&M Support Document for Chapter 10. Just more supporting notes I took while reading the chapter. Projection ideas, fact that can diagonalize for sure if all eigenvalues are different, examples of geo mult and alge mult not the same. Not much useful here. The last page discusses the idea that there are many congruences that can diagonalize symmetric A, but only a few will be orthogonal, ie, only a few will also be similarities.

Matrix Research. I went on a search and destroy mission to identify and prove more matrix theorems. Here we have **row echelon form**, row equivalence, **EROs**, finding rank from reduced form, matrix rep of the EROs, examples of ERO's. Then we have the **LU** and **PLU** factorization theorems. Next is **Sylvester's Law of Nullity**, which is a piece of work. More rank theorems. Then the whole **Householder** business and then the **QR factorization** (but not the "QR algorithm" which comes later). Then theorems about **banded matrices**, triangle form, generalized Hessenberg form UHn (my invention). Then we get into **polynomials with matrix coefficients**, leading to the **Cayley-Hamilton Theorem.** Things are then wrapped up with **Sylvester's Law of Inertia.**

Notes on "Numerical Recipes in Fortran 77". I downloaded this entire book, then printed and read just a few sections from its Chapter 11, see next binder section. The part that I perused discusses efficient numerical methods of finding eigenvalues and eigenvectors of a matrix. It first reviews the Jacobi method of grinding down with plane rotations. For symmetric, nowadays we do finites to tridiagonal form (Givens or Householder finites), then do the QR algorithm. For others, do balance then finites to get to Hessenberg form, then again do the QR algorithm.

Printed Chapter 11 sections of "Numerical Recipes in Fortran 77". See above comments, and see the printed table of contents for this book. This book is 1000 pages and addresses very many areas such as: linear algebra as I have already studied, Cholesky, sparse, Vandermonde, Toelplitz LU. All this just in Chapter 2. Then lots of Scheid-like topics: interpolation, integration, evaluation of functions, special functions, random numbers, sorting, finding roots, min max, then our Chapter 11 on eigenvalues, then FFT, statistics, modeling, doing numerical ODE's, boundary value problems, integral equations, PDE's. Of course this book also gives tested Fortran routines for doing all these things. They have similar books for C and C++ I think. A second volume uses the parallel constructs of Fortran 90.

The QR Factorization (Jerry Schultz). A nice little 4 pages of notes on this topic. This is where I first learned about the Householder vector and matrix and what it does.

The QR Algorithm. Nice overview paper by B. Parlett in the "top 10 algorithms" series, and also my notes on this paper.

Matrix Notes Section. This is a section of old notes on matrices that I had in one of my green math binders, I decided to move them all here. See index on first page of this section! A mixture of simple things and very high-powered theorems like Campbell-Hausdorf, direct sums and products. I had a great interest in all this stuff when I was studying group representations for SU(1,1) and such back in 1970's.

Comments on Diagonalization. These are all pre-Jordan form remarks, but still useful I think.

The Given's Method. I got it wrong in my Scheid notes, here it is done right I think.