Wilf "Mathematics for Physics Sciences" (1962) PhL 1.21.05

Jim Ball referenced this text for the Jacobi matrix business and perhaps other things. **Herbert Wilf** was in 1962 a math Prof at the U of Illinois (that was 43 years ago). The book is used for a 2-semester math Methods course for 1st and 2nd year grad students in physics, engineering or other non-math fields. I can only presume that this text was used at one time in Jim's "Math Methods" course which he taught on and off for 30 years in the U of Utah physics department.

Wilf is currently a math prof at U of Penn http://www.math.upenn.edu/FacData.html and has a personal web page with interesting stuff. This link gives a nice list of math books people have on line:
http://www.math.gatech.edu/%7Ecain/textbooks/onlinebooks.html
In his tribute to a mentor, he notes that the mentor really pushed the Jacobi matrix idea and he just recorded it in his book, so I guess this is something he is "famous" for.

The book has a very clean appearance, well laid out, good print font and organization, I like it. Bruce took it out from Marriott library for me, I have it for 30 days. Amazon used has one copy for about $20.

Chap 1: Vectors and Matrices. This of course contains some of the stuff I have done recently in my matrix binge. Vector space is defined, the $\mathcal{L}^2(a,b)$ example, Schwarz's inequality, norm, Fourier coefficient c_i, Parseval's Identity, Riemann-Lebesgue Lemma that $c_i \rightarrow 0$, linear independence, linear operators, Hermitian operators and eigenvalues. He uses A^* for adjoint (meaning transpose and CC), then $A=A^*$ is self-adjoint or Hermitian, basic facts about H operators, then unitary ones. Projection operators, then the world of NxN matrices. The cofactor^T is called the **adjugate** matrix (to avoid confusion with adjoint), uses overbar for CC. Gets inverse, assumes det product rule. Secular equation, diagonalization, similarity. The similarity that does the diagonalizing is called a **polar** matrix. Then functions of matrices, the C-H theorem, the companion matrix (whose secular is some desired poly). He refers to the idea of enlarging a matrix by one row and column (with our usual lines) as **bordering**. Positive definite matrices. Then on to rank and nullity. Simultaneous diagonalization. Numerical calc of eigenvalues. Matrices in ODE systems. Concept of irreducible matrix in the world of positive matrices (don't know about this one).

So this is all done in Sections like 1.5, and in theorem format like Theorem 1 through 34. Definitions are not bolded, they are just in-text. Refs and exericese. Householder appears as a ref but his matrix is not mentioned, maybe new in 1962. Scalar product is (a,b) throughout, very nice. As he says, he has to pick and choose what to include and what to omit. My review was I think more far ranging than this chapter, but what a great chapter! 45 pages.

Chap 2: Ortho Functions. (not called polys). Similar to Erdelyi, we have w(x) and (a,b) and ϕ_n. The thing $(x^n, x^m) = (1,x^{n+m}) = c_{n+m}$ is called **the moments matrix**, invertible. Uses k_n for lead coefficient. Shows things are polys, rules about the zeros. Then derives the recurrence condition using the B_n and C_n of Erdelyi, but does not use A_n and instead just writes $k_{n|+1}/k_n$. He shows that if your ϕ_n or orthonormal, then you get the symmetric (as I call it wrongly -- Jim form) α_n β_n form page 54. No mention of the monic form. Then on page 55 he writes the **Jacobi matrix** idea and shows that if x_i are the zeros, then you have an eigenvalue problem. This is the thing Jim is quoting and that you don't see very often. Notice bold for vector, as I do, and matrix as normal text but cap. Then the C-D identities. He does the weight shift idea that E quotes. Then comes a Rodriguez discussion that is a little more general than E, perhaps like Hildebrand's. Rules on zeros. Then Gauss Quadrature page 61. Shows the L function but does not call it Lagrange. Weights are called H_υ, zeros are x_i . Shows inversion formula with weight as integral of ϕ_n and ϕ_n' (he is using these instead of π in his L formula). Then on page 64 he gets the sum of poly^2

formula for the weights. So finally here it is, sitting in a book! Next is the classical polys with the standard Rodriguez form, uses G for E's X(x). Derives the ODE and then the diff formula. He has the same Leibniz' rule stuff to contend with. Then comes a section on specific polys and his spelling is **Tschebycheff**. Convergence of poly expansions, then some Fourier series work. Then **Fejer summability** (I never heard of that one) relating to F series.

So about 30 pages on this subject! This is of course another of the "topics" of my recent math review, which is why I am perusing the chapter in such detail. I complained earlier that none of my books had a section on this subject, well here is a book with such a section! But Erdelyi was OK for me.

Chap 3: Roots of Poly Equations. This deals with facts about the roots = zeros (mainly, about their location in the complex plane), not how to compute them. We have a Gauss-Lucas about location of roots in complex plane in convex hulls. Sturm sequences. Probably good stuff for filter theory. Nothing on poles here since just polys. An Erdos-Turan theorem! Newton sums. Chapter ends with Newton-Raphson.

So this is 24 pages I know little about, but now I know where to look!

Chap 4: Asymptotic Expansions. Here is *another* topic I reviewed recently. The O and o order symbols. A full 35 pages is expended on this subject, I suspect it is well done and I note it for later reference. Of course has Stirling in there somewhere.

Chap 5: ODEs. Yet another topic I went hunting for recently. General form of first order ODE as in Scheid. Discussion of Lipschitz condition and Picard's Theorem, dim things to me at the moment. Wintner's Method. Then on to numerical solving of ODEs. I don't see the phrase "difference equation" used, but it is here. Predictor corrector formulas. Stability. Then on page 166 we start into second order ODEs. Ordinary points and regular points. The Gamma function is studied from a strange def. Then Bessel functions from the ODE.

On numerical Scheid of course has more stuff. I complained I had no books with sections on ODE's, here is such a section. But of course it is too short to be really useful, just the basics in 44 pages.

Chap 6: Conformal Mapping. We did some of this in Jackson, and I will simply note its presence here. The name Montel appears. Then Schwarz-Christoffel mapping. Nice to have a section on this somewhere!

Chap 7: Extremum Problems. Lagrange Multipliers. Calculus of variations and Euler equation. Variational notation! Then we are into the Simplex Method (yet *another* recent subject of mine!). Then we are talking poly approximation of functions. Weierstrass theorem tells us poly fit can five uniform convergence over a range to any ε.

And boom, we are done! Then he seems to have solutions for ALL exercises in the book! Then a books bibliography followed by an original work biblio (eg, Dantzig on simplex). Index looks too short. Courant and Hilbert "Methods of Math Physics" , Erdelyi/Bateman gets a mention. Hildebrand has two books, one of which I have in Dover. Szego's 1939 book on ortho polys. Whittaker and Watson. It seems odd that I never attempted to build up a library of such books, they were always "too expensive".