This is obsolete but keep for a while.

(j) The Z-Transform Wiener-Khintchine Theorem

In FT Section 32 (c) we derive the continuum version of the Wiener-Khintchine theorem as follows, making use of the Convolution Theorem FT (3.6),

$$r_{\mathbf{x}}(t) = \int_{-\infty}^{\infty} dt'' x(t - t'') x(-t'') \quad . \qquad \text{// energy units} \qquad \text{FT (32.7)}$$

$$a(t) = \int_{-\infty}^{\infty} dt'' \, b(t-t'') \, c(t'') \qquad \Leftrightarrow \qquad A(\omega) = B(\omega) \, C(\omega) \quad . \qquad \text{FT (3.6)}$$

$$b(t) = x(t) \quad \leftrightarrow \quad B(\omega) = X(\omega)$$
$$c(t) = x(-t) \quad \leftrightarrow \quad C(\omega) = X(-\omega) = X(\omega)^* \qquad \text{// from FT (7.1) and (7.2)}$$

Thus, the diagonalized frequency-domain form $A(\omega) = B(\omega) \, C(\omega)$ is

$$R_{\mathbf{x}}(\omega) = |X(\omega)|^2 \, . \qquad \text{FT (32.8)}$$

Since

$$\mathcal{P}(\omega) \equiv \frac{\mathcal{E}(\omega)}{T} = \frac{|X(\omega)|^2}{2\pi T} \qquad \text{FT (33.23)}$$

where T is the duration of a pulse train, we find that

$$\mathcal{P}(\omega) = 2\pi T \, R_{\mathbf{x}}(\omega) \quad . \qquad \text{FT (34. 3)}$$

This says that the spectral power density of a pulse train can be obtained by computing the Fourier Integral Transform of the autocorrelation function and multiplying by $2\pi T$. We can regard this as the Wiener-Khintchine theorem, although strictly the theorem is that $R_{\mathbf{x}}(\omega) = |X(\omega)|^2$.

We shall now obtain a similar Wiener-Khintchine theorem for an infinite sequence which contains repeated P-length subsequences. We define the autocorrelation sequence as

$$r_{\mathbf{s}} \equiv \sum_{n=-\infty}^{\infty} y_{\mathbf{n}} \, y_{\mathbf{n+s}} = \sum_{n=-\infty}^{\infty} y_{\mathbf{-n}} \, y_{\mathbf{-n+s}}$$

where in the last expression we took $n \rightarrow -n$. We can then construct a Z Transform version of the Wiener-Khintchine theorem as follows:

$$r_{\mathbf{s}} = \sum_{n=-\infty}^{\infty} y_{\mathbf{s-n}} \, y_{\mathbf{-n}}$$

$$a_s = \sum_{n=-\infty}^{\infty} \Delta t \; b_{s-n} c_n \qquad \Leftrightarrow \qquad A''(z) = \Delta t \, B''(z) C''(z) \qquad \text{FT (24.5)}$$

$a_s = r_s \quad \leftrightarrow \quad A''(z) = R''(z)$
$b_k = y_k \quad \leftrightarrow \quad B''(z) = Y''(z)$
$c_k = y_{-k} \quad \leftrightarrow \quad C''(z) = Y''(z^{-1}) = Y''(z)^*$ // if the c_k are real
$\Delta t \quad \leftrightarrow \quad 1$

Thus, the diagonalized z-domain form $A''(z) = \Delta t \, B''(z) C''(z)$ is

$$R''(z) = Y''(z) \, Y''(z)^* = | Y''(z) |^2 \; .$$

It is shown in FT (34.14) that

$$\mathcal{P}(\omega) = T_1 \, \mathcal{P}_{pulse}(\omega) \, (1/T) \sum_{n=-\infty}^{\infty} \sum_{m=-\infty}^{\infty} y_m^* \, y_n \, e^{i\omega(m-n)T_1} \qquad \text{FT (34.14)}$$

which we easily rewrite as

$$\mathcal{P}(\omega) = \mathcal{P}_{pulse}(\omega) \, (T_1/T) \, | Y''(z) |^2 \qquad (2.5.43)$$

Using (2.5.42) we get,

$$\mathcal{P}(\omega) = \mathcal{P}_{pulse}(\omega) \, R''(z) \; . \qquad (2.5.44)$$

This says that the spectral power density of repeated P-subsequence infinite pulse train can be obtained by computing the Z Transform of the autocorrelation sequence and multiplying by $\mathcal{P}_{pulse}(\omega)$. This then is our Z Transform version of the Wiener-Khintchine theorem.