This has been installed.

Operation of the Figure 7 Polynomial Multiplier

As we have seen, the polynomial multiplier of Figure 7 generates a product of this form

$$O(z) = z^{-k} H(z) I(z)$$

Recall that the input polynomial, output polynomial, and divisor polynomial have these forms, where for each we put the highest power of z on the left:

$$I(z) = i_0 + i_1 z^{-1} + i_2 z^{-2} + \ldots\ldots$$
$$O(z) = o_0 + o_1 z^{-1} + o_2 z^{-2} + \ldots\ldots$$
$$H(z) = h_3 z^3 + h_2 z^2 + h_1 z + h_0 \quad .$$

Although one can draw pictures for general k, things are very much clearer of we take a specific k value, so consider the following k=3 case of Figure 7. Recall that the registers are all initialized to zero.

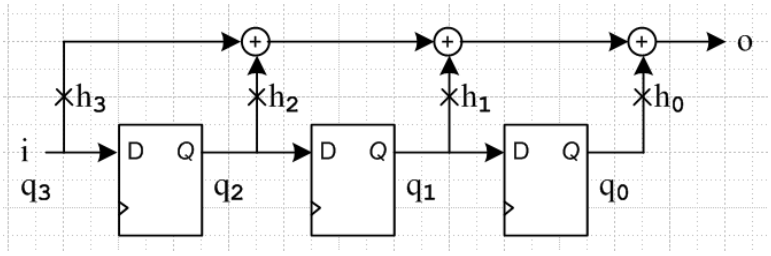


Figure 7 Multiplier with k = 3

To demonstrate the operation, we first write out a bunch of symbols, then explain them below:

$$\begin{array}{l}
\qquad h_3z^3 + h_2z^2 + h_1z^1 + h_0z^0 \\
\ldots\ldots + i_3z^{-3} + i_2z^{-2} + i_1z^{-1} + i_0z^0 \\
\text{---------------------------------------} \\
\qquad i_0h_3z^3 + i_0h_2z^2 + i_0h_1z^1 + i_0h_0z^0 \\
\qquad\qquad i_1h_3z^2 + i_1h_2z^1 + i_1h_1z^0 + i_1h_0z^{-1} \\
\qquad\qquad\qquad i_2h_3z^1 + i_2h_2z^0 + i_2h_1z^{-1} + i_2h_0z^{-2} \\
\qquad\qquad\qquad\qquad i_3h_3z^0 + i_3h_2z^{-1} + i_3h_1z^{-2} + i_3h_0z^{-3} \\
\qquad\qquad\qquad\qquad\qquad i_4h_3z^{-1} + i_4h_2z^{-2} + i_4h_1z^{-3} + i_4h_0z^{-4} \\
\qquad\qquad\qquad\qquad\qquad\qquad i_5h_3z^{-2} + i_5h_2z^{-3} + i_5h_1z^{-4} + i_5h_0z^{-5} \\
\qquad\qquad\qquad\qquad\qquad\qquad\qquad **** + ***** + ***** + \ldots \\
\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad ***** + ***** + \ldots \\
\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad ***** + \ldots \\
\text{---} \\
\qquad o_0z^3 + o_1z^2 + o_2z^1 + o_3z^0 + o_4z^{-1} + o_5z^{-2} + o_6z^{-3} + o_7z^{-4} + o_8z^{-5} \ldots
\end{array}$$

$$= z^3 [\, o_0 z^0 + o_1 z^{-1} + o_2 z^{-2} + o_3 z^{-3} + o_4 z^{-4} + o_5 z^{-5} + o_6 z^{-6} + o_7 z^{-7} + o_8 z^{-8} \ldots]$$

First of all, when multiplying two polynomials by hand, one is certainly allowed to arbitrarily order the powers in each polynomial as one wants. In the above we have H(z) with its highest power on the left, but we show I(z) with its highest power on the right. We then mechanically do the multiplication as we were taught in grade school. The rightmost element $i_0 z^0$ of I(z) is multiplied by H(z) as shown to get the first row. Then the next multiplier element $i_1 z^{-1}$ of I(z) is multiplied by H(z) as shown to get the second row, and so on. The rows are aligned so that powers of z match in each column. Normally these rows march to the left, but because I(z) has negative powers, they march to the right.

After all the rows are written out, we *add up the columns* to get the overall product, as shown under the second dotted line. Since $z^3 O(z) = H(z)\, I(z)$, we factor out z^3 on the very last line to expose O(z) in the bracket. The idea here is that, for example, $o_2 = i_0 h_1 + i_1 h_2 + i_2 h_3$. To get each o_j symbol, we have to add up the column of expressions above it. This column addition is performed by the set of adders appearing in Figure 7!

We can associate a clock tick with each column going left to right. Each clock tick causes another input symbol to shift into the left end of the shift register, and causes all the symbols already in the register to shift to the right one position. At clock tick 0 the output is $o_0 = i_0 h_3$. At clock tick 1 the output is $o_1 = i_0 h_2 + i_1 h_3$, and so on. By clock tick 3, the registers contain (i_3, i_2, i_1, i_0) and all three adders are "active" to produce a sum of four numbers as shown in the column above o_3. As time moves on, the three adders continue to add four symbols of a column. If the input polynomial is finite so the last non-zero input is i_r and all subsequent inputs are 0, the rows taper off the way they began. For example, if i_5 were the last input, the above picture applies if we delete all the rows with asterisks. In this case, looking at the two polynomials being multiplied, it is clear that the largest power in the result is z^3 and the smallest is z^{-5}, so there will be 3-(-5)+1 = 9 symbols in the product, namely o_0 through o_8. In the general case where the last input is i_r and H(z) has degree k, the product will have k-(-r)+1 = k+r+1 symbols.

Not needed:

$$\begin{aligned}
&i_0 h_3 z^3 + i_0 h_2 z^2 + i_0 h_1 z^1 + i_0 h_0 z^0 \\
&\quad i_1 h_3 z^2 + i_1 h_2 z^1 + i_1 h_1 z^0 + i_1 h_0 z^{-1} \\
&\quad\quad i_2 h_3 z^1 + i_2 h_2 z^0 + i_2 h_1 z^{-1} + i_2 h_0 z^{-2} \\
&\quad\quad\quad i_3 h_3 z^0 + i_3 h_2 z^{-1} + i_3 h_1 z^{-2} + i_3 h_0 z^{-3} \\
&\quad\quad\quad\quad i_4 h_3 z^{-1} + i_4 h_2 z^{-2} + i_4 h_1 z^{-3} + i_4 h_0 z^{-4} \\
&\quad\quad\quad\quad\quad i_5 h_3 z^{-2} + i_5 h_2 z^{-3} + i_5 h_1 z^{-4} + i_5 h_0 z^{-5} \\
&\text{----------} \\
&o_0 z^3 + o_1 z^2 + o_2 z^1 + o_3 z^0 + o_4 z^{-1} + o_5 z^{-2} + o_6 z^{-3} + o_7 z^{-4} + o_8 z^{-5}
\end{aligned}$$

$$= z^3 [\, o_0 z^0 + o_1 z^{-1} + o_2 z^{-2} + o_3 z^{-3} + o_4 z^{-4} + o_5 z^{-5} + o_6 z^{-6} + o_7 z^{-7} + o_8 z^{-8}]$$

****

[---------- H(z) ----------]

$$\ldots + i_3z^{-3} + i_2z^{-2} + i_1z^{-1} + i_0z^0$$

---

i_0[---------- H(z) ----------]

$i_1 z^{-1}$[---------- H(z) ----------]

$i_2 z^{-2}$[---------- H(z) ----------]

$i_3 z^{-3}$[---------- H(z) ----------]

$i_4 z^{-4}$[---------- H(z) ----------]

$i_5 z^{-5}$[---------- H(z) ----------]

$$o_0z^3 + o_1z^2 + o_2z^1 + o_3z^0 + o_4z^{-1} + o_5z^{-2} + o_6z^{-3} + o_7z^{-4} + o_8z^{-5} \ldots$$

$$= z^3 [\, o_0z^0 + o_1z^{-1} + o_2z^{-2} + o_3z^{-3} + o_4z^{-4} + o_5z^{-5} + o_6z^{-6} + o_7z^{-7} + o_8z^{-8} \ldots]$$

Operation of the Figure 8 Polynomial Multiplier

Again we consider the case k = 3 and this time we assume a finite input stream (i_0, i_1, i_2, i_3, i_4).

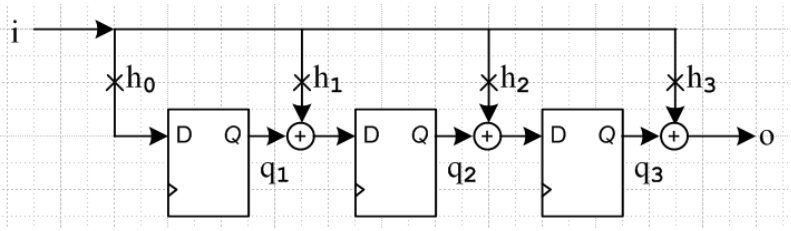


Figure 8 Multiplier with k = 3

We first write down the math, and then explain it below:

$$h_0z^0 + h_1z^1 + h_2z^2 + h_3z^3$$
$$i_4z^{-3} + i_3z^{-3} + i_2z^{-2} + i_1z^{-1} + i_0z^0$$

---

partial product #0 $\quad i_0h_0z^0 + i_0h_1z^1 + i_0h_2z^2 + i_0h_3z^3$
$i_1h_0z^{-1} + i_1h_1z^0 + i_1h_2z^1 + i_1h_3z^2$

---

partial product #1 $\quad i_1h_0z^{-1} + a_0z^0 + a_1z^1 + a_2z^2 + i_0h_3z^3$
$i_2h_0z^{-2} + i_2h_1z^{-1} + i_2h_2z^0 + i_2h_3z^1$

---

partial product #2 $\quad i_2h_0z^{-2} + b_{-1}z^{-1} + b_0z^0 + b_1z^1 + a_2z^2 + i_0h_3z^3$
$i_3h_0z^{-3} + i_3h_1z^{-2} + i_3h_2z^{-1} + i_3h_3z^0$

---

etc. $\quad i_3h_0z^{-3} + c_{-2}z^{-2} + c_{-1}z^{-1} + c_0z^0 + b_1z^1 + a_2z^2 + i_0h_3z^3$
$i_4h_0z^{-4} + i_4h_1z^{-3} + i_4h_2z^{-2} + i_4h_3z^{-1}$

$$i_4h_0z^{-4} + d_{-3}z^{-3} + d_{-2}z^{-2} + d_{-1}z^{-1} + c_0z^0 + b_1z^1 + a_2z^2 + i_0h_3z^3$$

$$
\begin{array}{l}
\qquad\quad 0 \quad\; 0 \quad\;\; 0 \quad\;\; 0 \quad \text{(since } i_5 = 0\text{)} \\
\hline
\qquad\quad 0 \quad i_4h_0z^{-4} + d_{-3}z^{-3} + d_{-2}z^{-2} + d_{-1}z^{-1} + c_0z^0 + b_1z^1 + a_2z^2 + i_0h_3z^3 \\
\quad 0 \quad 0 \quad 0 \quad\;\; 0 \quad \text{(since } i_6 = 0\text{)} \\
\hline
\quad\; 0 \quad 0 \quad i_4h_0z^{-4} + d_{-3}z^{-3} + d_{-2}z^{-2} + d_{-1}z^{-1} + c_0z^0 + b_1z^1 + a_2z^2 + i_0h_3z^3 \\
0 \quad 0 \quad 0 \quad 0 \quad \text{(since } i_7 = 0\text{)} \\
\hline
0 \quad 0 \quad 0 \quad i_4h_0z^{-4} + d_{-3}z^{-3} + d_{-2}z^{-2} + d_{-1}z^{-1} + c_0z^0 + b_1z^1 + a_2z^2 + i_0h_3z^3
\end{array}
$$

This time, both the multiplicand H(z) and the multiplier I(z) are written with the largest power on the right. We then carry out the usual grade school multiplication method as shown which yields a sequence of partial products. For example, partial product #0 is the product if we account only for i_0's contribution to the full product. Then partial product #1 is the full product if only i_0 and i_1 are accounted for.

As we go down, we make up names for coefficients. For example, $a_0 \equiv i_0h_0 + i_1h_1$ and later $b_0 \equiv a_0 + i_2h_2$. When i_0 is at the input, the output is $o_0 = i_0h_3$ as shown in red on the partial product #0 line.

In each partial product, the three expressions shown in green are the contents of the three registers q_1, q_2, q_3 and we could regard the expression in red as the output o captured by some register not shown off to the right. The black expression underneath each partial product represents the other set of inputs to the array of adders in Fig 7. On each clock, the result of an addition is strobed into the registers (green), an output in red is strobed out off the o bus, and the blue expressions are no longer stored anywhere since we don't need them.

The first output (ignoring powers of z) is $o_0 = i_0h_3$ and the second is $o_1 = a_2 = i_0h_0 + i_1h_1$, both in agreement with the Figure 8 circuit analysis.

Since i_4 is assumed to be the last non-zero input symbol, the machinery keeps running in its endgame with all-zero lines added in, and during this phase the three register contents are just shifted right to produce the last 3 output symbols, out after which the output goes to zero.

$$
\begin{array}{l}
\qquad\quad h_3z^3 + h_2z^2 + h_1z + h_0z^0 \\
\ldots + i_3z^{-3} + i_2z^{-2} + i_1z^{-1} + i_0z^0 \\
\hline
\qquad i_0h_3z^3 + i_0h_2z^2 + i_0h_1z^1 + i_0h_0z^0 \\
\qquad\quad\; i_1h_3z^2 + i_1h_2z^1 + i_1h_1z^0 + i_1h_0z^{-1} \\
\hline
\qquad i_0h_3z^3 + a_1z^2 + a_2z^1 + a_3z^0 + i_1h_0z^{-1}
\end{array}
$$

$$
\begin{array}{l}
\qquad\qquad\qquad\qquad i_2h_3z^1 + i_2h_2z^0 + i_2h_1z^{-1} + i_2h_0z^{-2} \\
\qquad\qquad\qquad\quad\; i_3h_3z^0 + i_3h_2z^{-1} + i_3h_1z^{-2} + i_3h_0z^{-3} \\
\qquad\qquad\qquad\quad\; i_4h_3z^{-1} + i_4h_2z^{-2} + i_4h_1z^{-3} + i_4h_0z^{-4} \\
\qquad\qquad\qquad\quad\; i_5h_3z^{-2} + i_5h_2z^{-3} + i_5h_1z^{-4} + i_5h_0z^{-5} \\
\qquad\qquad\qquad\qquad\quad\; **** + ***** + **** \ldots \\
\qquad\qquad\qquad\qquad\qquad\; ***** + **** \ldots
\end{array}
$$

$$\text{*****...}$$

$$o_0z^3 + o_1z^2 + o_2z^1 + o_3z^0 + o_4z^{-1} + o_5z^{-2} + o_6z^{-3} + o_7z^{-4} + o_8z^{-5} ..$$

$$= z^3 [\, o_0z^0 + o_1z^{-1} + o_2z^{-2} + o_3z^{-3} + o_4z^{-4} + o_5z^{-5} + o_6z^{-6} + o_7z^{-7} + o_8z^{-8} .. \,]$$

Operation of the Figure 2 Polynomial Divider

Again we consider the case k = 3:

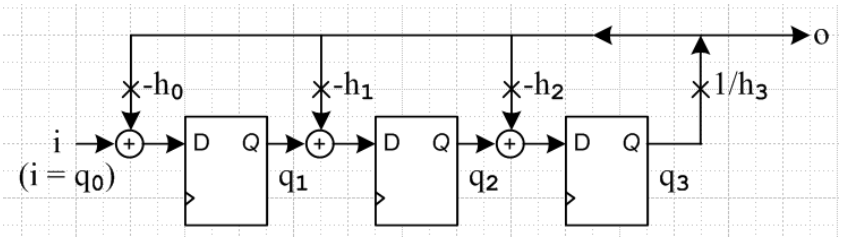


Figure 2: The standard polynomial divider circuit with k = 3.

To demonstrate the operation, we first write out a bunch of symbols, then explain them below:

$$\begin{array}{l} \qquad\qquad\qquad\qquad o_3z^{-3} + o_4z^{-4} + o_5z^{-5} + o_6z^{-6} + o_7z^{-7} + o_8z^{-8} + ... \\ h_3z^3 + h_2z^2 + h_1z^1 + h_0z^0 \;|\; i_0z^0 + i_1z^{-1} + i_2z^{-2} + i_3z^{-3} + i_4z^{-4} + i_5z^{-5} + ... \\ \qquad\qquad\quad - o_3h_3z^0 - o_3h_2z^{-1} - o_3h_1z^{-2} - o_3h_0z^{-3} \\ \qquad\qquad\quad \text{---} \end{array}$$

Current Dividend #1

$$\begin{array}{l} \qquad\qquad\quad a_1z^{-1} + a_2z^{-2} + a_3z^{-3} + i_4z^{-4} + i_5z^{-5} + ... \\ \qquad\quad - o_4h_3z^{-1} - o_4h_2z^{-2} - o_4h_1z^{-3} - o_4h_0z^{-4} \\ \qquad\quad \text{---} \end{array}$$

Current Dividend #2

$$\begin{array}{l} \qquad\qquad\quad b_2z^{-2} + b_3z^{-3} + b_4z^{-4} + i_5z^{-5} + ... \\ \qquad\quad - o_5h_3z^{-2} - o_5h_2z^{-3} - o_5h_1z^{-4} - o_5h_0z^{-5} \\ \qquad\quad \text{---} \end{array}$$

Current Dividend #3

$$\begin{array}{l} \qquad\qquad\quad c_3z^{-3} + c_4z^{-4} + c_5z^{-5} + ... \\ \qquad\quad - o_6h_3z^{-3} - o_6h_2z^{-4} - o_6h_1z^{-5} - o_6h_0z^{-6} \\ \qquad\quad \text{---------------------------------------} \end{array}$$

and on forever

The divisor H(z) and dividend I(z) are written in the usual manner with the highest power to the left. We then perform the usual grade school long division process. In every vertical column, the powers of z match. The original dividend appears under the radical (current dividend #0). At each stage of the long division process we have a new "current dividend" as shown. The first three terms of each current dividend are shown in green.

To give things a compact look, we define a lot of new symbols along the way, in this order (down each column, then to the next column):

$o_3 \equiv i_0/h_3$ $\qquad$ $o_4 \equiv a_1/h_3$ $\qquad$ $o_5 \equiv b_2/h_3$ $\qquad$ $o_6 \equiv c_3/h_3$

$a_1 \equiv i_1 - o_3h_2$ $\quad b_2 \equiv a_2 - o_4h_2$ $\quad c_3 \equiv b_3 - o_5h_2$ $\quad$ ***

$a_2 \equiv i_2 - o_3h_1$ $\quad b_3 \equiv a_3 - o_4h_1$ $\quad c_4 \equiv a_3 - o_5h_1$ $\quad$ ***

$a_3 \equiv i_3 - o_3h_0$ $\quad b_4 \equiv i_4 - o_4h_0$ $\quad c_5 \equiv i_4 - o_5h_0$ $\quad$ ***

and so on. Notice how the output sequence o_3, o_4, o_5 ..... is being computed in the first row of this little table. The three green terms (without the z powers) indicate the contents of the three registers, but in the order (q_3, q_2, q_1) which is backwards from the ordering in Figure 2. Looking at the sequence of symbol definitions above, one realizes that only the green register contents are necessary to compute all the output coefficients o_n.

It is assumed as usual that the shift register is cleared before i_0 comes in. The input symbols just shift right in and appear as (i_2, i_1, i_0) in registers (q_1, q_2, q_3). This is so because up to this point there is no feedback on the o bus. These register contents are indicated by $i_0z^0 + i_1z^{-1} + i_2z^{-2}$ in the original dividend, and as just noted, the order is reversed. After this point, the feedback is activated, and things progress as shown above. Even if the input sequence truncates after some i_r, the output sequence in general continues forever. The exception of course is the case that I(z) is an exact multiple of H(z).

Operation of the Figure 1 Polynomial Divider

Again we consider the case k = 3:

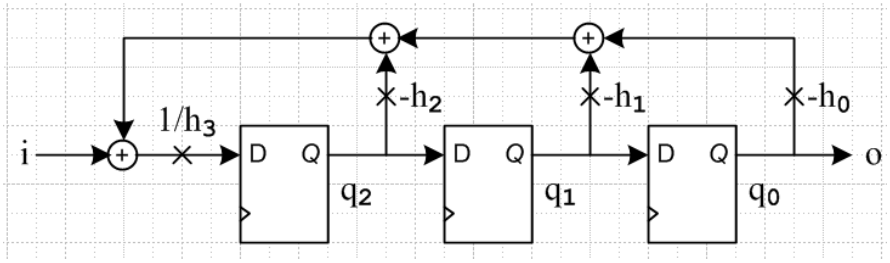


Figure 1: The scrambler style polynomial divider circuit with k = 3.

We assume the registers are pre-cleared. When input i_0 appears on the i bus, the D input to register q_2 is then (i_0/h_3) and on the next clock edge this becomes the contents of q_2. In two more clocks, this value will appear in q_0 so (i_0/h_3) must be o_3 ! In fact, at any instant in time, the three registers always hold the next three o_j outputs since the registers form a simple shift register. This fact is crucial to understanding how this circuit works. We repeat the above long division layout but with different "coloration":

$$\begin{array}{rl} & o_3z^{-3} + o_4z^{-4} + o_5z^{-5} + o_6z^{-6} + o_7z^{-7} + o_8z^{-8} + \ldots. \\ h_3z^3 + h_2z^2 + h_1z^1 + h_0z^0 \;|\; & i_0z^0 + i_1z^{-1} + i_2z^{-2} + i_3z^{-3} + i_4z^{-4} + i_5z^{-5} + \ldots.. \\ & -o_3h_3z^0 - o_3h_2z^{-1} - o_3h_1z^{-2} - o_3h_0z^{-3} \\ & \text{---} \end{array}$$

Current Dividend #1

$$\begin{array}{rl} & a_1z^{-1} + a_2z^{-2} + a_3z^{-3} + i_4z^{-4} + i_5z^{-5} + \ldots.. \\ & -o_4h_3z^{-1} - o_4h_2z^{-2} - o_4h_1z^{-3} - o_4h_0z^{-4} \\ & \text{---} \end{array}$$

Current Dividend #2

$$\begin{array}{rl} & b_2z^{-2} + b_3z^{-3} + b_4z^{-4} + i_5z^{-5} + \ldots. \\ & -o_5h_3z^{-2} - o_5h_2z^{-3} - o_5h_1z^{-4} - o_5h_0z^{-5} \end{array}$$

Current Dividend #3

$$c_3z^{-3} + c_4z^{-4} + c_5z^{-5} + \ldots.$$
$$-o_6h_3z^{-3} - o_6h_2z^{-4} - o_6h_1z^{-5} - o_6h_0z^{-6}$$

Current Dividend #4

$$d_4z^{-4} + d_5z^{-5} + d_6z^{-6} + \ldots.$$
$$-o_7h_3z^{-4} - o_7h_2z^{-5} - o_7h_1z^{-6} \ldots.$$

Current Dividend #5

$$e_5z^{-5} + e_6z^{-6} + \ldots$$
$$-o_8h_3z^{-5} - o_8h_2z^{-6}$$

Let's start in the middle of things to see what the adders in Figure 1 are doing. Consider the column of red expressions which has i_4 at the top. At this time, the output is o_4 and the input is i_4. The adders are therefore computing the following sum at this time (based on the fact just stated about outputs-to-be)

D input to $q_2 = (1/h_3) * [i_4 + (-h_0)o_4 + (-h_1)o_5 + (-h_2)o_6]$

where the expressions inside [...] match those of the red column just noted. On the next clock edge, this value is strobed into register q_2 and it is then destined to become o_7, which is shown in blue underneath the red column of expressions just noted. With the previous Figure 2 circuit, this same sum was in effect computed, but in a set of steps associated with the current dividends. Here the sum is computed in a single shot.

If we look one clock later, we have the next column of red expressions being added to determine o_8. And one clock earlier, we have a column adding up expressions to determine o_6. Before this time, some of the adder inputs are 0 so the column of red expressions added has fewer than 4 elements.

****

To demonstrate the operation, we first write out a bunch of symbols, then explain them below:

$$(i_0/h_3)z^{-3}+(b_1/h_3)z^{-4} + (d_2/h_3)z^{-5} + (f_3/h_3)z^{-6} + (j_4/h_3)z^{-7}.$$

$$h_3z^3 + h_2z^2 + h_1z^1 + h_0z^0 \;\Big|\; i_0z^0 + i_1z^{-1} + i_2z^{-2} + i_3z^{-3} + i_3z^{-4}$$
$$- i_0z^0 - (i_0/h_3)h_2z^{-1} - (i_0/h_3)h_1z^{-2} - (i_0/h_3)h_0z^{-3}$$

$$b_1z^{-1} + b_2z^{-2} + b_3z^{-3} + b_4z^{-4}$$
$$- b_1z^{-1} - c_2z^{-2} - c_3z^{-3} - c_4z^{-4}$$

$$d_2z^{-2} + d_3z^{-3} + d_4z^{-4}$$
$$- d_2z^{-2} - e_3z^{-3} - e_4z^{-4} - e_5z^{-5}$$

$$f_3z^{-3} + f_4z^{-4} + f_5z^{-5}$$
$$-f_3z^{-3} - g_4z^{-4} - g_5z^{-5} - g_6z^{-6}$$

$$
\begin{array}{r}
\text{----------} \\
j_4z^{-4} + j_5z^{-5} + \ldots \\
-j_4z^{-4} - k_5z^{-5} + \ldots \\
\text{----------} \\
m_5z^{-5} + \ldots\ldots \\
-m_5z^{-5} - \ldots\ldots \\
\text{----------}
\end{array}
$$

and on forever

$$
\begin{array}{r}
o_3z^{-3} + o_4z^{-4} + o_5z^{-5} + o_6z^{-6} + o_7z^{-7} + o_8z^{-8} + \ldots. \\
\hline
h_3z^3 + h_2z^2 + h_1z^1 + h_0z^0 \;|\; i_0z^0 + i_1z^{-1} + i_2z^{-2} + i_3z^{-3} + i_4z^{-4} + i_5z^{-5} + \ldots\ldots \\
-o_3h_3z^0 - o_3h_2z^{-1} - o_3h_1z^{-2} - o_3h_0z^{-3} \\
-o_4h_3z^{-1} - o_4h_2z^{-2} - o_4h_1z^{-3} - o_4h_0z^{-4} \\
-o_5h_3z^{-2} - o_5h_2z^{-3} - o_5h_1z^{-4} - o_5h_0z^{-5} \\
-o_6h_3z^{-3} - o_6h_2z^{-4} - o_6h_1z^{-5} - o_6h_0z^{-6} \\
\text{----------}
\end{array}
$$

and on forever

$$
\begin{array}{r}
(i_0/h_3)z^{-3}+(b_1/h_3)z^{-4}+(d_2/h_3)z^{-5}+(f_3/h_3)z^{-6}+(j_4/h_3)z^{-7}+(m_5/h_3)z^{-8}+\ldots \\
\hline
h_3z^3 + h_2z^2 + h_1z^1 + h_0z^0 \;|\; i_0z^0 + i_1z^{-1} + i_2z^{-2} + i_3z^{-3} + i_3z^{-4} \\
-i_0z^0 - a_1z^{-1} - a_2z^{-2} - a_3z^{-3} \\
\text{----------}
\end{array}
$$

$$
\begin{array}{l}
\quad b_1 z^{-1} + b_2 z^{-2} + b_3 z^{-3} + b_4 z^{-4} \\
-\,b_1 z^{-1} - c_2 z^{-2} - c_3 z^{-3} - c_4 z^{-4} \\
\hline
\qquad\quad d_2 z^{-2} + d_3 z^{-3} + d_4 z^{-4} \\
\qquad -\,d_2 z^{-2} - e_3 z^{-3} - e_4 z^{-4} - e_5 z^{-5} \\
\hline
\qquad\qquad\quad f_3 z^{-3} + f_4 z^{-4} + f_5 z^{-5} \\
\qquad\qquad -\,f_3 z^{-3} - g_4 z^{-4} - g_5 z^{-5} - g_6 z^{-6} \\
\hline
\qquad\qquad\qquad\quad j_4 z^{-4} + j_5 z^{-5} + \ldots \\
\qquad\qquad\qquad -\,j_4 z^{-4} - k_5 z^{-5} + \ldots \\
\hline
\qquad\qquad\qquad\qquad\quad m_5 z^{-5} + \ldots\ldots \\
\qquad\qquad\qquad\qquad -\,m_5 z^{-5} - \ldots\ldots \\
\hline
\qquad\qquad\qquad\qquad\qquad \text{and on forever}
\end{array}
$$

The divisor and dividend are written in the usual manner with the highest power to the left. We then perform the usual grade school long division process. To give things a compact look, we define a lot of new symbols along the way, For example:

$a_1 \equiv (i_0/h_3)h_2$	$b_1 \equiv i_1 - a_1$	$c_2 \equiv (b_1/h_3)h_2$	$d_2 \equiv b_2 - c_2$	and so on
$a_2 \equiv (i_0/h_3)h_1$	$b_2 \equiv i_2 - a_2$	$c_3 \equiv (b_1/h_3)h_1$	$d_3 \equiv b_3 - c_3$	
$a_3 \equiv (i_0/h_3)h_0$	$b_3 \equiv i_3 - a_3$	$c_4 \equiv (b_1/h_3)h_0$	$d_4 \equiv b_4 - c_4$	
	$b_4 \equiv i_3$			

In general, even with a finite dividend as in this example, the quotient sequence goes on forever.