Summary of docs in this folder PhL 1.19.11

These summaries are in chrono order by creation date. The general subject is convincing myself that Jim's first and second proposed solutions L to L([1/r]) = $2\pi\delta^2(\mathbf{r})$ really work. These are in-house docs not ever sent to Jim.

1. Jim's First Proposed L (9 pages, 1.8.11) **1**

2. Find L in 1D $L_x s(x,y)=\delta(x-y)$ (18 pages, 1.9.11) **1**

3. Find L in 1D Lr(x) = δ(x) for the Love kernel. (10 pages, 1.10.11) **1**

4. Dissing a claim I made in my jim email of 1_13_11 5PM.doc (8 pages, 1.14.11) **2**

1. Jim's First Proposed L (9 pages, 1.8.11)

Jim made this proposal by phone (which is his way to comm) on 1/8 and I then wrote this in-house doc pondering his proposal which eventually became $-\partial_{\mathbf{r}}^2(r[1/r])$. Section 1 was written perhaps on 1/17 after I knew more things, so is really out of place here. In Section 1 I use the Stak theorem to verify Jim's first proposal, but I did not know how to do this on 1/8. In Section 2 (back on 1/8) I ponder how one would prove the usual 3D fact without using the divergence theorem, and I don't know! Then in Section 3 I look at that Stak theorem and I search for a sequence, trying [1/r] = 1/(r+ε) and then trying [1/r] = $1/r^{1+\varepsilon}$, both acted upon by Jim's proposed L, but neither works. So I am unable to find a Stakgold sequence that works on 1/8.

2. Find L in 1D $L_x s(x,y)=\delta(x-y)$ (18 pages, 1.9.11)

Here I first consider this problem for the *general* 1D kernel k(x,y), which I call s(x,y). My method of attack is always to expand both sides using a Fourier transform expansion. In **Section 1** I first try this with $a_{\mathbf{n}}(x)$ coefficient functions and I get a horrible mess. Then in part (e) I try again with constant coefficients $a_{\mathbf{n}}$ and I learn that there is no hope (in the "local sense") unless s(x,y) = r(x-y), but even in that case if r(x) is even, there exists no solution with constant coefficients. I later repeat this argument in **Section 6** and show how it applies to the Love kernel. In **Section 2** I try a double Fourier on s(x,y) and this goes nowhere. In **Section 3** I try starting with the r(x-y) form and using $a_{\mathbf{n}}(x)$ and get a horrible mess. In **Section 4** I try r(x-y) = |x-y| because I know this has to work with $L = \partial_{\mathbf{x}}^2$ since it is the 1D fundy solution, but this r(x) has no Fourier Transform! In **Section 5** I think of Lr = δ as a matrix problem, and the problem is simple of you use basis functions $\varphi_{\mathbf{n}}(x)$ which are EFs of L. But this is a useless method, since we know neither $L_{\mathbf{x}}$ nor $\varphi_{\mathbf{n}}(x)$. At this point I sent my first email to R Price.

3. Find L in 1D Lr(x) = δ(x) for the Love kernel. (10 pages, 1.10.11)

I show that, for the Love integral equation's kernel, r(x) = $(d/\pi) / [\, d^2 + x^2 \,]$, there exists no differential operator $L_{\mathbf{x}}$ *with constant coefficients* (even if we allow infinite order) such that $L_{\mathbf{x}}\, r(x) = \delta(x)$. I show this two different ways and arrive at the same conclusion twice. It follows that there is no $L_{\mathbf{x}}$ such that $L_{\mathbf{x}}\, r(x-y) = \delta(x-y)$ and thus it follows that the Love integral equation cannot be represented as a differential equation of the form

$$L_x[f(x) - 1] = \mp \int_{-1}^{1} dt\, L_x\, r(x-t)\, f(t) \;=\; \mp \int_{-1}^{1} dt\, L_x\, \delta(x-t)\, f(t) \;=\; \mp\, f(x)\, \theta(x>-1)\theta(x<+1)$$

where L_x has the form just stated. To some extent this analysis is replicated in the previous doc above. The main thing is that the Love FT is $e^{-d|k|}$ which is not analytic in k at k=0.

4. Dissing a claim I made in my jim email of 1_13_11 5PM.doc (8 pages, 1.14.11)

When I wrote this doc, I was pretty much convinced (and in agreement with Jim Ball) that Jim's second proposal L= (1/r)∂(r [1/r]) was NOT true. But I had a lot of evidence that it WAS true. So here I am trying to show why each of my three pieces of evidence was wrong.

In **Section 1** I examined the Stakgold Theorem n=1 proof of (2.11). But this proof shows conclusively that Jim #2 works! For a while I had a factor of 1/2 discrepancy, but I then learned (Section 4) about this factor which arises only in the n=1 case.

In **Section 2** I examine the Stakgold Theorem itself. I review how one proves the three facts he calls (a),(b),(c) on page 13 of his book. I convince myself it is correct and there is nothing special about my application which should make it not be correct. In fact g(r) can blow up at r=0, it is only the normalization condition that matters.

In **Section 3** I convince myself that the correct n=1 normalization condition has 1/2 on the RHS.

In **Section 4** I explain away the factor of 1/2 discrepancy I was getting in Section 1.

By **Section 5** I have finally realized that Jim #2 really does work, and I list off the pieces of evidence.

In **Section 6** I do for Jim another Stakgold n=2 case $\lim_{\varepsilon\to 0}\, [\, \varepsilon / (r^2+\varepsilon^2)^{3/2}\,] = \delta^2(\mathbf{r})$. I prove it here, and then I note that it appears in Stakgold as an exercise. I think Jim felt that you really needed some derivatives ∂_r (and preferably two as in $\partial_r{}^2$) in order to construct a limit expression which makes a delta function, and this Section 6 case is a counterexample. I think Jim is a little amazed by these limit forms, and I don't think despite 30 years of teaching Math Methods that he ever ran into them before in the form Stakgold gives.