Jim, let's start all over and everything is stated here only in Cartesian coordinates. 1.11.11

Question #1: What Cartesian 3D differential operator L_{xyz} makes this equation be true:

$$L_{xyz}\left(1/\sqrt{(x-x')^2+ (y-y')^2 + (z-z')^2} \right) = \delta(x-x')\delta(y-y')\delta(z-z')$$

Answer:

$$L_{xyz} = - (1/4\pi) (\partial_x^2 + \partial_y^2 + \partial_z^2)$$

Question #2: What Cartesian 2D differential operator L_{xy} makes this equation be true:

$$L_{xy}\left(1/\sqrt{(x-x')^2+ (y-y')^2} \right) = \delta(x-x')\delta(y-y')$$

Answer:

$L_{xy} =$

Please put your Cartesian-coordinates answer in the box provided above.

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I don't know the answer. It might be that no operator L_{xy} exists which solves Problem #2. Or maybe it is some horrible thing like this, with horrible coefficient functions,

$$L_{xy} = a(x,y)\partial_x + a(y,x)\partial_y + b(x,y)\,\partial_x^2 + b(y,x)\,\partial_y^2 + c(x,y)\partial_x\partial_y + \text{lots of higher terms.}$$

I am pretty sure this is NOT the answer

$$L_{xy} = \partial_x^2 + \partial_y^2$$

The reason is that I know that

$$(\partial_x^2 + \partial_y^2)\left[\ln\left(\sqrt{(x-x')^2+ (y-y')^2}\right) \right] = \text{constant} * \delta(x-x')\delta(y-y')$$

and it seems unlikely that two different functions would solve this equation.