

## *Table of Fourier Transform Pairs*

| Function, $f(t)$                                                                                                                  | Fourier Transform, $F(\omega)$                                                                         |
|-----------------------------------------------------------------------------------------------------------------------------------|--------------------------------------------------------------------------------------------------------|
| <i>Definition of Inverse Fourier Transform</i><br>$f(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} F(\omega) e^{j\omega t} d\omega$ | <i>Definition of Fourier Transform</i><br>$F(\omega) = \int_{-\infty}^{\infty} f(t) e^{-j\omega t} dt$ |
| $f(t - t_0)$                                                                                                                      | $F(\omega) e^{-j\omega t_0}$                                                                           |
| $f(t) e^{j\omega_0 t}$                                                                                                            | $F(\omega - \omega_0)$                                                                                 |
| $f(\alpha t)$                                                                                                                     | $\frac{1}{ \alpha } F\left(\frac{\omega}{\alpha}\right)$                                               |
| $F(t)$                                                                                                                            | $2\pi f(-\omega)$                                                                                      |
| $\frac{d^n f(t)}{dt^n}$                                                                                                           | $(j\omega)^n F(\omega)$                                                                                |
| $(-jt)^n f(t)$                                                                                                                    | $\frac{d^n F(\omega)}{d\omega^n}$                                                                      |
| $\int_{-\infty}^t f(\tau) d\tau$                                                                                                  | $\frac{F(\omega)}{j\omega} + \pi F(0) \delta(\omega)$                                                  |
| $\delta(t)$                                                                                                                       | 1                                                                                                      |
| $e^{j\omega_0 t}$                                                                                                                 | $2\pi \delta(\omega - \omega_0)$                                                                       |
| $\text{sgn}(t)$                                                                                                                   | $\frac{2}{j\omega}$                                                                                    |

# Fourier Transform Table

UBC M267 Resources for 2005

| $F(t)$                                                                                     | $\widehat{F}(\omega)$                                           | Notes (0)                                                 |
|--------------------------------------------------------------------------------------------|-----------------------------------------------------------------|-----------------------------------------------------------|
| $f(t)$                                                                                     | $\int_{-\infty}^{\infty} f(t)e^{-i\omega t} dt$                 | Definition. (1)                                           |
| $\frac{1}{2\pi} \int_{-\infty}^{\infty} \widehat{f}(\omega)e^{i\omega t} d\omega$          | $\widehat{f}(\omega)$                                           | Inversion formula. (2)                                    |
| $\widehat{f}(-t)$                                                                          | $2\pi f(\omega)$                                                | Duality property. (3)                                     |
| $e^{-at}u(t)$                                                                              | $\frac{1}{a + i\omega}$                                         | $a$ constant, $\Re(a) > 0$ (4)                            |
| $e^{-a t }$                                                                                | $\frac{2a}{a^2 + \omega^2}$                                     | $a$ constant, $\Re(a) > 0$ (5)                            |
| $\beta(t) = \begin{cases} 1, & \text{if }  t  < 1, \\ 0, & \text{if }  t  > 1 \end{cases}$ | $2 \operatorname{sinc}(\omega) = 2 \frac{\sin(\omega)}{\omega}$ | Boxcar in time. (6)                                       |
| $\frac{1}{\pi} \operatorname{sinc}(t)$                                                     | $\beta(\omega)$                                                 | Boxcar in frequency. (7)                                  |
| $f'(t)$                                                                                    | $i\omega \widehat{f}(\omega)$                                   | Derivative in time. (8)                                   |
| $f''(t)$                                                                                   | $(i\omega)^2 \widehat{f}(\omega)$                               | Higher derivatives similar. (9)                           |
| $tf(t)$                                                                                    | $i \frac{d}{d\omega} \widehat{f}(\omega)$                       | Derivative in frequency. (10)                             |
| $t^2 f(t)$                                                                                 | $i^2 \frac{d^2}{d\omega^2} \widehat{f}(\omega)$                 | Higher derivatives similar. (11)                          |
| $e^{i\omega_0 t} f(t)$                                                                     | $\widehat{f}(\omega - \omega_0)$                                | Modulation property. (12)                                 |
| $f\left(\frac{t - t_0}{k}\right)$                                                          | $ke^{-i\omega t_0} \widehat{f}(k\omega)$                        | Time shift and squeeze. (13)                              |
| $(f * g)(t)$                                                                               | $\widehat{f}(\omega)\widehat{g}(\omega)$                        | Convolution in time. (14)                                 |
| $u(t) = \begin{cases} 0, & \text{if } t < 0 \\ 1, & \text{if } t > 0 \end{cases}$          | $\frac{1}{i\omega} + \pi\delta(\omega)$                         | Heaviside step function. (15)                             |
| $\delta(t - t_0)f(t)$                                                                      | $e^{-i\omega t_0} f(t_0)$                                       | Assumes $f$ continuous at $t_0$ . (16)                    |
| $e^{i\omega_0 t}$                                                                          | $2\pi\delta(\omega - \omega_0)$                                 | Useful for $\sin(\omega_0 t)$ , $\cos(\omega_0 t)$ . (17) |

**Convolution:** 
$$(f * g)(t) = \int_{-\infty}^{\infty} f(t - u)g(u) du = \int_{-\infty}^{\infty} f(u)g(t - u) du.$$

**Parseval:** 
$$\int_{-\infty}^{\infty} |f(t)|^2 dt = \frac{1}{2\pi} \int_{-\infty}^{\infty} |\widehat{f}(\omega)|^2 d\omega.$$

|                                                            |                                                                                                                   |
|------------------------------------------------------------|-------------------------------------------------------------------------------------------------------------------|
| $j \frac{1}{\pi t}$                                        | $\text{sgn}(\omega)$                                                                                              |
| $u(t)$                                                     | $\pi \delta(\omega) + \frac{1}{j\omega}$                                                                          |
| $\sum_{n=-\infty}^{\infty} F_n e^{jn\omega_0 t}$           | $2\pi \sum_{n=-\infty}^{\infty} F_n \delta(\omega - n\omega_0)$                                                   |
| $\text{rect}(\frac{t}{\tau})$                              | $\tau \text{Sa}(\frac{\omega \tau}{2})$                                                                           |
| $\frac{B}{2\pi} \text{Sa}(\frac{Bt}{2})$                   | $\text{rect}(\frac{\omega}{B})$                                                                                   |
| $\text{tri}(t)$                                            | $\text{Sa}^2(\frac{\omega}{2})$                                                                                   |
| $A \cos(\frac{\pi t}{2\tau}) \text{rect}(\frac{t}{2\tau})$ | $\frac{A\pi}{\tau} \frac{\cos(\omega \tau)}{(\pi/2\tau)^2 - \omega^2}$                                            |
| $\cos(\omega_0 t)$                                         | $\pi [\delta(\omega - \omega_0) + \delta(\omega + \omega_0)]$                                                     |
| $\sin(\omega_0 t)$                                         | $\frac{\pi}{j} [\delta(\omega - \omega_0) - \delta(\omega + \omega_0)]$                                           |
| $u(t) \cos(\omega_0 t)$                                    | $\frac{\pi}{2} [\delta(\omega - \omega_0) + \delta(\omega + \omega_0)] + \frac{j\omega}{\omega_0^2 - \omega^2}$   |
| $u(t) \sin(\omega_0 t)$                                    | $\frac{\pi}{2j} [\delta(\omega - \omega_0) - \delta(\omega + \omega_0)] + \frac{\omega^2}{\omega_0^2 - \omega^2}$ |
| $u(t) e^{-\alpha t} \cos(\omega_0 t)$                      | $\frac{(\alpha + j\omega)}{\omega_0^2 + (\alpha + j\omega)^2}$                                                    |

|                                      |                                                      |
|--------------------------------------|------------------------------------------------------|
| $u(t)e^{-\alpha t} \sin(\omega_0 t)$ | $\frac{\omega_0}{\omega_0^2 + (\alpha + j\omega)^2}$ |
| $e^{-\alpha t }$                     | $\frac{2\alpha}{\alpha^2 + \omega^2}$                |
| $e^{-t^2/(2\sigma^2)}$               | $\sigma\sqrt{2\pi} e^{-\sigma^2\omega^2/2}$          |
| $u(t)e^{-\alpha t}$                  | $\frac{1}{\alpha + j\omega}$                         |
| $u(t)te^{-\alpha t}$                 | $\frac{1}{(\alpha + j\omega)^2}$                     |

➤ **Trigonometric Fourier Series**

$$f(t) = a_0 + \sum_{n=1}^{\infty} (a_n \cos(\omega_0 nt) + b_n \sin(\omega_0 nt))$$

where

$$a_0 = \frac{1}{T} \int_0^T f(t) dt, \quad a_n = \frac{2}{T} \int_0^T f(t) \cos(\omega_0 nt) dt, \text{ and}$$

$$b_n = \frac{2}{T} \int_0^T f(t) \sin(\omega_0 nt) dt$$

➤ **Complex Exponential Fourier Series**

$$f(t) = \sum_{n=-\infty}^{\infty} F_n e^{j\omega_0 nt}, \text{ where } F_n = \frac{1}{T} \int_0^T f(t) e^{-j\omega_0 nt} dt$$

### ***Some Useful Mathematical Relationships***

|                                                       |
|-------------------------------------------------------|
| $\cos(x) = \frac{e^{jx} + e^{-jx}}{2}$                |
| $\sin(x) = \frac{e^{jx} - e^{-jx}}{2j}$               |
| $\cos(x \pm y) = \cos(x) \cos(y) \mp \sin(x) \sin(y)$ |
| $\sin(x \pm y) = \sin(x) \cos(y) \pm \cos(x) \sin(y)$ |
| $\cos(2x) = \cos^2(x) - \sin^2(x)$                    |
| $\sin(2x) = 2 \sin(x) \cos(x)$                        |
| $2 \cos^2(x) = 1 + \cos(2x)$                          |
| $2 \sin^2(x) = 1 - \cos(2x)$                          |
| $\cos^2(x) + \sin^2(x) = 1$                           |
| $2 \cos(x) \cos(y) = \cos(x - y) + \cos(x + y)$       |
| $2 \sin(x) \sin(y) = \cos(x - y) - \cos(x + y)$       |
| $2 \sin(x) \cos(y) = \sin(x - y) + \sin(x + y)$       |

### *Useful Integrals*

|                                          |                                                                                             |
|------------------------------------------|---------------------------------------------------------------------------------------------|
| $\int \cos(x) dx$                        | $\sin(x)$                                                                                   |
| $\int \sin(x) dx$                        | $-\cos(x)$                                                                                  |
| $\int x \cos(x) dx$                      | $\cos(x) + x \sin(x)$                                                                       |
| $\int x \sin(x) dx$                      | $\sin(x) - x \cos(x)$                                                                       |
| $\int x^2 \cos(x) dx$                    | $2x \cos(x) + (x^2 - 2) \sin(x)$                                                            |
| $\int x^2 \sin(x) dx$                    | $2x \sin(x) - (x^2 - 2) \cos(x)$                                                            |
| $\int e^{\alpha x} dx$                   | $\frac{e^{\alpha x}}{\alpha}$                                                               |
| $\int x e^{\alpha x} dx$                 | $e^{\alpha x} \left[ \frac{x}{\alpha} - \frac{1}{\alpha^2} \right]$                         |
| $\int x^2 e^{\alpha x} dx$               | $e^{\alpha x} \left[ \frac{x^2}{\alpha} - \frac{2x}{\alpha^2} + \frac{2}{\alpha^3} \right]$ |
| $\int \frac{dx}{\alpha + \beta x}$       | $\frac{1}{\beta} \ln \alpha + \beta x $                                                     |
| $\int \frac{dx}{\alpha^2 + \beta^2 x^2}$ | $\frac{1}{\alpha\beta} \tan^{-1}\left(\frac{\beta x}{\alpha}\right)$                        |

# Engineering Tables/Fourier Transform Table 2

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| Signal<br>$g(t) \equiv$                                                         | Fourier transform<br>unitary, angular frequency<br>$G(\omega) \equiv$             | Fourier transform<br>unitary, ordinary frequency<br>$G(f) \equiv$ | Remarks                                                                                                                                               |
|---------------------------------------------------------------------------------|-----------------------------------------------------------------------------------|-------------------------------------------------------------------|-------------------------------------------------------------------------------------------------------------------------------------------------------|
| $\frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} G(\omega) e^{i\omega t} d\omega$ | $\frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} g(t) e^{-i\omega t} dt$            | $\int_{-\infty}^{\infty} g(t) e^{-i2\pi f t} dt$                  |                                                                                                                                                       |
| 10 $\text{rect}(at)$                                                            | $\frac{1}{\sqrt{2\pi a^2}} \cdot \text{sinc}\left(\frac{\omega}{2\pi a}\right)$   | $\frac{1}{ a } \cdot \text{sinc}\left(\frac{f}{a}\right)$         | The rectangular pulse and the normalized sinc function                                                                                                |
| 11 $\text{sinc}(at)$                                                            | $\frac{1}{\sqrt{2\pi a^2}} \cdot \text{rect}\left(\frac{\omega}{2\pi a}\right)$   | $\frac{1}{ a } \cdot \text{rect}\left(\frac{f}{a}\right)$         | Dual of rule 10. The rectangular function is an idealized low-pass filter, and the sinc function is the non-causal impulse response of such a filter. |
| 12 $\text{sinc}^2(at)$                                                          | $\frac{1}{\sqrt{2\pi a^2}} \cdot \text{tri}\left(\frac{\omega}{2\pi a}\right)$    | $\frac{1}{ a } \cdot \text{tri}\left(\frac{f}{a}\right)$          | $\text{tri}$ is the triangular function                                                                                                               |
| 13 $\text{tri}(at)$                                                             | $\frac{1}{\sqrt{2\pi a^2}} \cdot \text{sinc}^2\left(\frac{\omega}{2\pi a}\right)$ | $\frac{1}{ a } \cdot \text{sinc}^2\left(\frac{f}{a}\right)$       | Dual of rule 12.                                                                                                                                      |
| 14 $e^{-\alpha t^2}$                                                            | $\frac{1}{\sqrt{2\alpha}} \cdot e^{-\frac{\omega^2}{4\alpha}}$                    | $\sqrt{\frac{\pi}{\alpha}} \cdot e^{-\frac{(\pi f)^2}{\alpha}}$   | Shows that the Gaussian function $\exp(-at^2)$ is its own Fourier transform. For this to be integrable we must have $\text{Re}(a) > 0$ .              |

|                                                  |                                                                                                                                         |                                                                                                 |                                                                                                            |
|--------------------------------------------------|-----------------------------------------------------------------------------------------------------------------------------------------|-------------------------------------------------------------------------------------------------|------------------------------------------------------------------------------------------------------------|
| $e^{iat^2} = e^{-\alpha t^2} \Big _{\alpha=-ia}$ | $\frac{1}{\sqrt{2a}} \cdot e^{-i\left(\frac{\omega^2}{4a} - \frac{\pi}{4}\right)}$                                                      | $\sqrt{\frac{\pi}{a}} \cdot e^{-i\left(\frac{\pi^2 f^2}{a} - \frac{\pi}{4}\right)}$             | common in optics                                                                                           |
| $\cos(at^2)$                                     | $\frac{1}{\sqrt{2a}} \cos\left(\frac{\omega^2}{4a} - \frac{\pi}{4}\right)$                                                              | $\sqrt{\frac{\pi}{a}} \cos\left(\frac{\pi^2 f^2}{a} - \frac{\pi}{4}\right)$                     |                                                                                                            |
| $\sin(at^2)$                                     | $\frac{-1}{\sqrt{2a}} \sin\left(\frac{\omega^2}{4a} - \frac{\pi}{4}\right)$                                                             | $-\sqrt{\frac{\pi}{a}} \sin\left(\frac{\pi^2 f^2}{a} - \frac{\pi}{4}\right)$                    |                                                                                                            |
| $e^{-a t }$                                      | $\sqrt{\frac{2}{\pi}} \cdot \frac{a}{a^2 + \omega^2}$                                                                                   | $\frac{2a}{a^2 + 4\pi^2 f^2}$                                                                   | $a > 0$                                                                                                    |
| $\frac{1}{\sqrt{ t }}$                           | $\frac{1}{\sqrt{ \omega }}$                                                                                                             | $\frac{1}{\sqrt{ f }}$                                                                          | the transform is the function itself                                                                       |
| $J_0(t)$                                         | $\sqrt{\frac{2}{\pi}} \cdot \frac{\text{rect}\left(\frac{\omega}{2}\right)}{\sqrt{1 - \omega^2}}$                                       | $\frac{2 \cdot \text{rect}(\pi f)}{\sqrt{1 - 4\pi^2 f^2}}$                                      | $J_0(t)$ is the Bessel function of first kind of order 0, <i>rect</i> is the rectangular function          |
| $J_n(t)$                                         | $\sqrt{\frac{2}{\pi}} \frac{(-i)^n T_n(\omega) \text{rect}\left(\frac{\omega}{2}\right)}{\sqrt{1 - \omega^2}}$                          | $\frac{2(-i)^n T_n(2\pi f) \text{rect}(\pi f)}{\sqrt{1 - 4\pi^2 f^2}}$                          | it's the generalization of the previous transform; $T_n(t)$ is the Chebyshev polynomial of the first kind. |
| $\frac{J_n(t)}{t}$                               | $\sqrt{\frac{2}{\pi}} \frac{i}{n} (-i)^n \cdot U_{n-1}(\omega)$<br>$\cdot \sqrt{1 - \omega^2} \text{rect}\left(\frac{\omega}{2}\right)$ | $\frac{2i}{n} (-i)^n \cdot U_{n-1}(2\pi f)$<br>$\cdot \sqrt{1 - 4\pi^2 f^2} \text{rect}(\pi f)$ | $U_n(t)$ is the Chebyshev polynomial of the second kind                                                    |

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