This was installed in App F.

$$r_s \equiv \lim_{N\to\infty}\left[\frac{1}{(2N+1)}\sum_{n=-N}^{N} y_n\, y_{n+s}\right] = \langle y_n\, y_{n+s}\rangle_1 . \qquad (32.16) \qquad \text{(F.41a)}$$

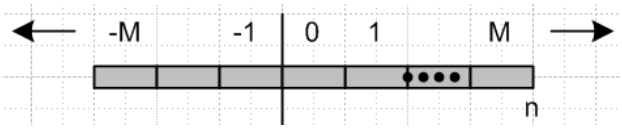
For large N, we can replace the sum endpoints by -N = -MP and +N = MP ≈ (M+1)P where M is also large. The sum then has (2M+1)P terms, so

$$r_s \equiv \lim_{M\to\infty}\left[\frac{1}{(2M+1)}\frac{1}{P}\sum_{n=-MP}^{(M+1)P} y_n\, y_{n+s}\right] .$$

We now let n = IP + n' where n' takes values 0,1..P-1 and I = Int(n/P). Then the single sum above can be written,

$$\sum_{n=-MP}^{(M+1)P} y_n\, y_{n+s} = \sum_{I=-M}^{M}\sum_{n'=0}^{P-1} y_{IP+n'}\, y_{IP+n'+s}$$

where this drawing shows how the sum is now a double sum where I denotes segments containing P points, and n' counts the points in a segment,



We then process this sum using the periodicity (F.5) to get

$$= \sum_{I=-M}^{M}\sum_{n'=0}^{P-1} y_{n'}\, y_{n'+s} = \left(\sum_{I=-M}^{M} 1\right)\left(\sum_{n=0}^{P-1} y_n\, y_{n+s}\right) = (2M+1)\left(\sum_{n=0}^{P-1} y_n\, y_{n+s}\right) .$$

Inserting the expression into r_s, we get this alternate form for r_s

$$r_s = \lim_{M\to\infty}\left[\frac{1}{(2M+1)}\frac{1}{P}\left\{(2M+1)\left(\sum_{n=0}^{P-1} y_n\, y_{n+s}\right)\right\}\right]$$

or

$$r_s = \frac{1}{P}\sum_{n=0}^{P-1} y_n\, y_{n+s} = \langle y_n\, y_{n+s}\rangle_1 \qquad \text{(F.41b)}$$