temp storage PhL 7.29.13

Here I am just storing an old and ill-formed App G section on sequences.

(g) Experiments with Sequences

Our pulse trains have a set of amplitudes $\{y_i\}$ which form a sequence. If the sequence has N elements, then a sequence can be written $\{y_1, y_2.....y_N\}$. When many pulse trains are generated in some line code (perhaps AMI), one finds that the values of y_n in position n of the sequence can vary, and that there is some probability distribution associated with the parameter y_n. An experiment could be the generation of such a sequence by some apparatus according to some set of rules (those for an AMI line code, say), and after many experiments one could find a probability distribution for parameter y_n. Thus, the position n in the sequence is associated with a random variable we must call Y_n which takes values y_n.

This notation leads to some confusion. When we had a random variable H which took values h, we might have enumerated the set of h values of the sample space as $\{h_k\}$! To maintain clarity, if we have random variable Y_n which takes values y_n, we should enumerate those values as $(y_n)_k$. Here, y_n is the name of a parameter, just as h was the name of a parameter. Thus, to write down a specific sequence "i" we really should say

$\{(y_1)_i, (y_2)_i.....(y_N)_i\}$ = sequence "i"

or

$\{y_1^{(i)}, y_2^{(i)}..... y_2^{(i)}\}$ = sequence "i"

where $(y_2)_i = y_2^{(i)}$ is some specific symbol value, perhaps $(y_2)_i = y_2^{(i)} = 5$. We used the latter notation when talking about an ensemble of pulse trains in Section ***.

On the other hand if we want to represent an arbitrary sequence whose elements are *variables*, we would write the sequence as

$\{y_1, y_2.....y_N\}$.

In our first experiment, our apparatus makes a single sequence we will call sequence 1 " : $\{(y_1)_1, (y_2)_1\}$. The elements of this sequence are the amplitudes of a very short pulse train. The symbols are taken from the finite set {1,2,3,4,5,6}. The experiment creates values for variables y_1 and y_2 according to some probabilistic algorithm so there is some $pmf_{Y1}(y_1)$ and some $pmf_{Y2}(y_2)$. Then Y_1 and Y_2 are random variables, and so is $Z = Y_1Y_2$. There will be some $pmf_Z(z) = \Sigma_{y1,y2}\ pmf_{Y1Y2}(y_1,y_2)$.

We now interrupt this experiment to get another fact on the table:

Fact:

The expected value of

y_1y_2 resulting from running this experiment many times will be

$$\langle y_1y_2\rangle = \Sigma_{i=1}^{6}\Sigma_{j=1}^{6}\, y_iy_j \quad \text{pmf}_{Y1Y2}(y_i,y_j) \ .$$

$$\langle y_1y_2\rangle = \Sigma_{i=1}^{6}\Sigma_{j=1}^{6}\, y_iy_j \quad \text{pmf}(y_1 = y_i, y_2 = y_j) \ .$$

In the case that Y_1 and Y_2 are uncorrelated and $\text{pmf}_{Y1}(w) = \text{pmf}_{Y2}(w) = 1/6$, a plot of $\text{pmf}_Z(z)$ for $z = y_1y_2$ will look exactly like the last plot shown above. The expected value of y_1y_2 will be

$$\langle y_1y_2\rangle = \Sigma_{i=1}^{6}\Sigma_{j=1}^{6}\, y_iy_j\, \text{pmf}_{Y1}(y_i)\, \text{pmf}_{Y2}(y_j) = (1/36)\,\Sigma_{i=1}^{6}\Sigma_{j=1}^{6}\ y_iy_j$$

$$= \ (1/36)(\,\Sigma_{i=1}^{6} y_i)^2 = \ (1/36)\ \ (1 + 2 + 3.. + 6)^2 = \ (1/36)\ (21)^2 = 49/4 = 12.25$$

Alternatively, using the p(z) values which make up $\text{pmf}_Z(z)$ shown above,

$$\langle y_1y_2\rangle = \Sigma_{k=1}^{36}\, z\ p(z) = \quad \texttt{sum(z * (p(z)/36), z = 1..36);}$$

$$\frac{49}{4}$$

In a different experiment with the same pulse train we can study the random variable Y_1^2 which relates to just the first position in the pulse train. The sample space of possible outcomes is {1,4,9,16,25,36} . We find in general that, for the simple case,

$$\langle y_1^2\rangle \ = \Sigma_{i=1}^{6}\, y_i^2\ \text{pmf}_{Y1}(y_i) \ = (1/6)\,\Sigma_{i=1}^{6}\, y_i^2 \ = (1/6)(\ 1+4+9+16+25+36\ ) = 91/6 = 15.17$$

and the same for $\langle y_2^2\rangle$.

In our second experiment, we consider an arbitrary pulse train with amplitudes $\{y_1,y_2.....y_N\}$ that is created by our apparatus with some $\text{pdf}_{Y1Y2Y3...YN}(y_1,y_2.....y_N)$. For some particular pulse train positions i and j we have the following expected value,

$$\langle y_iy_j\rangle = \ \Sigma_{a=1}^{6}\Sigma_{b=1}^{6}\ .....\ \ y_iy_j\ \text{pdf}_{Y1Y2Y3...YN}(a,b...i...j...)$$

where the indices a,b,c.... sum over all the y_k positions in $\text{pdf}_{Y1Y2Y3...YN}(y_1,y_2.....y_N)$ other than y_i and y_j.

TBC. Notice that when you run this experiment a million times, you create an ensemble of pulse trains, so I have finally arrived at a place I have been looking for. Also

$$\langle y_i^2\rangle = \ \Sigma_{a=1}^{6}\Sigma_{b=1}^{6}\ .....\ \ y_iy_j\ \text{pdf}_{Y1Y2Y3...YN}(a,b...i...i...)$$

If everything is independent, then

$\langle y_i y_j \rangle = \Sigma_{a=1}^{6} \Sigma_{b=1}^{6} \ldots\ldots\ y_i y_j \, pdf(a)pdf(b)....pdf(y_i)...pdf(y_j),,,$

$= y_i y_j . pdf(y_i) . pdf(y_j)$

$\langle y_i^2 \rangle = y_i^2 \, pdf(y_i) . pdf(y_i)$ // by same argument

This is for a particular position, seems strange there is no sum!