Rewrite of FT Section 34 (b) PhL 7.22.13

Summary: I am looking for some place to *use* my digital W-K R"(z) thing. The *statement* of this digital W-K gets inserted *earlier* in Sec 32. One page only below. Maybe we can delete this file. I did end up wedging the W-K R"(z) usage into this section 34 (b) which I think is the logical place it should appear, see FT doc for how it turned out.

(b) Spectral power density for a General Pulse Train

In Section 33 (d) we dealt with simple pulse trains. Here we consider the more general amplitude modulated pulse train. In all equations, one can replace $\sum_{n=-\infty}^{\infty}$ by $\sum_{n=-N}^{N}$ to adapt the equation to a finite pulse train instead of an infinite one. We start then with

$$x(t) = \sum_{n=-\infty}^{\infty} y_n \; x_{\mathbf{pulse}}(t - t_n) \qquad \textit{(33.1)} \qquad (34.5)$$

$$X(\omega) = (1/T_1) X_{\mathbf{pulse}}(\omega)\, X'(\omega) = X_{\mathbf{pulse}}(\omega)\, X''(z) \qquad \textit{(25.3)} \qquad (34.6)$$

$$X''(z) = X'(\omega)/T_1 = \sum_{n=-\infty}^{\infty} y_n \, e^{-i\omega n T_1} = Y''(z) \qquad \textit{(24.2)} \qquad (34.7)$$

Comment:

To find the frequency-domain power spectrum of a signal x(t), our first task is to compute $|\, X(\omega)\,|^2$. From (34.6) we get

$$|X(\omega)|^2 = |X_{\mathbf{pulse}}(\omega)|^2 \, (1/T_1)^2 \, |X'(\omega)|^2 = |X_{\mathbf{pulse}}(\omega)|^2 \, |\, X''(z)\,|^2 \qquad z = e^{i\omega T_1} \qquad (34.8)$$

We therefore must deal with the following object, using (34.7),

$$|\, X''(z)\,|^2 = (1/T_1)^2 \, |X'(\omega)|^2 = \Big|\, \sum_{n=-\infty}^{\infty} y_n \, e^{-i\omega n T_1} \,\Big|^2 = |\, Y''(z)\,|^2$$

$$= \sum_{n=-\infty}^{\infty} y_n \, e^{-i\omega n T_1} \sum_{m=-\infty}^{\infty} y_m^* \, e^{+i\omega m T_1} = \sum_{n=-\infty}^{\infty} \sum_{m=-\infty}^{\infty} y_m^* \, y_n \, e^{i\omega (m-n) T_1} \qquad (34.9)$$

so that

$$|\, X(\omega)\,|^2 = |\, X_{\mathbf{pulse}}(\omega)\,|^2 \sum_{n=-\infty}^{\infty} \sum_{m=-\infty}^{\infty} y_m^* \, y_n \, e^{i\omega (m-n) T_1} \quad . \qquad (34.10)$$

From box (34.4) we then have

$$\mathcal{E}(\omega) \equiv |X(\omega)|^2/2\pi \;=\; (1/2\pi)\,|\,X_{\mathbf{pulse}}(\omega)\,|^2 \sum_{n=-\infty}^{\infty} \sum_{m=-\infty}^{\infty} y_m^{*}\, y_n\, e^{i\omega(m-n)T_1} \tag{34.11}$$

$$\mathcal{P}(\omega) \;\equiv \frac{\mathcal{E}(\omega)}{T} \;= (1/2\pi T)\,|\,X_{\mathbf{pulse}}(\omega)\,|^2 \sum_{n=-\infty}^{\infty} \sum_{m=-\infty}^{\infty} y_m^{*}\, y_n\, e^{i\omega(m-n)T_1} \tag{34.12}$$

which we can write as (again using box (34.4) results)

$$\mathcal{E}(\omega) = \; T_1\, \mathcal{P}_{\mathbf{pulse}}(\omega) \sum_{n=-\infty}^{\infty} \sum_{m=-\infty}^{\infty} y_m^{*}\, y_n\, e^{i\omega(m-n)T_1} \tag{34.13}$$

$$\mathcal{P}(\omega) \;= T_1\, \mathcal{P}_{\mathbf{pulse}}(\omega)\,(1/T) \sum_{n=-\infty}^{\infty} \sum_{m=-\infty}^{\infty} y_m^{*}\, y_n\, e^{i\omega(m-n)T_1} \tag{34.14}$$

$$P \;=\; \int_{-\infty}^{\infty} d\omega\; \mathcal{P}(\omega) \quad . \qquad \textit{(33.31)} \tag{34.15}$$

Not knowing details of the y_n there is not much else we can do in these expressions.