Repairs PhL 7.22.13

I have wrongly referred to Y'(z) and Y"(z) as X'(z) and X"(z). I reviewed all the places this happened and I have now installed all the repairs! This was a bug in the originally released FT. The bug existed in Sections 33 and 34 as noted below. I think this is all OK.

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33. Spectral power density of a Simple Pulse Train

In subsections (a) through (d) we deal only with simple pulse trains. Along the way, certain facts are developed which apply to general as well as simple pulse trains. In subsection (e) we gather together these general facts, and then show how quantities of interest can be related to the autocorrelation function.

Comment: Some texts use the phrase "power spectral density" (PSD). Although this wins on Google by a ratio of 3 to 1, we still prefer the phrase "spectral power density", and the same for "spectral energy density".

(a) Infinite Simple Pulse Train

This is the first section in which we use the δ(0) notation of Appendix A which may make the reader feel a bit uncomfortable. In subsection (b) we shall repeat everything for a finite pulse train and then take the limit N→∞ to obtain the same results without using δ(0). In both sections we shall include the Z Transform in passing, but our main work is in the ω variable, not the z variable.

From Section 14 (a) we know the spectrum of an infinite pulse train formed from pulses $x_{pulse}(t)$ separated by time T_1 ,

$$x(t) = \sum_{n=-\infty}^{\infty} x_{pulse}(t - nT_1) \qquad (14.1)$$

$$X(\omega) = X_{pulse}(\omega) \sum_{m=-\infty}^{\infty} 2\pi\ \delta(\omega T_1 - 2\pi m)\ . \qquad (14.4)$$

We can obtain the same expressions from box (25.4) which summarizes amplitude modulated pulse trains by setting all amplitudes to $y_n = 1$,

$$x(t) = \sum_{n=-\infty}^{\infty} y_n\ x_{pulse}(t - t_n) = \sum_{n=-\infty}^{\infty} x_{pulse}(t - t_n) \qquad (33.1)$$

$$X(\omega) = (1/T_1)X_{pulse}(\omega)\ Y'(\omega) = X_{pulse}(\omega)\ Y''(z) \qquad (33.2)$$

$$Y''(z) = Y'(\omega)/T_1 = \sum_{n=-\infty}^{\infty} y_n\ e^{-i\omega n T_1} = \sum_{n=-\infty}^{\infty} e^{-i\omega n T_1}\ . \qquad (33.3)$$

Y'(ω) is the Digital Fourier Transform of $y_n = 1$, and Y''(z) is the Z Transform, where $z = e^{i\omega T_1}$.

In the last line we then use (13.2)

$$\sum_{n=-\infty}^{\infty} e^{\pm ink} = \sum_{m=-\infty}^{\infty} 2\pi\delta(k - 2\pi m) \qquad -\infty < k < \infty \qquad (13.2)$$

so that (33.3) becomes

$$Y''(z) = Y'(\omega)/T_1 = \sum_{n=-\infty}^{\infty} e^{-i\omega n T_1} = \sum_{m=-\infty}^{\infty} 2\pi\delta(\omega T_1 - 2\pi m) \qquad (33.4)$$

and then (33.2) says

$$X(\omega) = (1/T_1)X_{pulse}(\omega)\, Y'(\omega) = X_{pulse}(\omega) \sum_{m=-\infty}^{\infty} 2\pi\delta(\omega T_1 - 2\pi m) \qquad (33.5)$$

in agreement with (14.4) quoted just above (33.1).

To find the power spectrum of a signal x(t), our first task is to compute $| X(\omega) |^2$. From (33.2) we get

$$|X(\omega)|^2 = |X_{pulse}(\omega)|^2\, (1/T_1)^2\, |Y'(\omega)|^2 = |X_{pulse}(\omega)|^2\, |\, Y''(z)\, |^2 \quad . \qquad z = e^{i\omega T_1} \qquad (33.6)$$

We therefore must deal with the following object, using (33.4),

$$|\, Y''(z)\, |^2 = (1/T_1)^2\, |Y'(\omega)|^2 = \Big[\sum_{m=-\infty}^{\infty} 2\pi\delta(\omega T_1 - 2\pi m) \Big]^2 \quad , \qquad (33.7)$$

and we are now faced with the issue of squaring delta functions. Formally these objects don't exist in the realm of distribution theory, but (as discussed in Appendix A) we can deal with them in an *ad hoc* way which proves to be useful. Consider,

$$\Big[\sum_{m=-\infty}^{\infty} 2\pi\delta(\omega T_1 - 2\pi m) \Big]^2 = \sum_{m=-\infty}^{\infty} 2\pi\delta(\omega T_1 - 2\pi m) \sum_{n=-\infty}^{\infty} 2\pi\delta(\omega T_1 - 2\pi n)$$

$$= \sum_{m=-\infty}^{\infty} \sum_{n=-\infty}^{\infty} 2\pi\delta(\omega T_1 - 2\pi m)\, 2\pi\delta(\omega T_1 - 2\pi n) \, .$$

Looking at the product of the two delta functions, there can be no contribution to the double sum unless m = n, so we continue

$$= \sum_{m=-\infty}^{\infty} 2\pi\delta(\omega T_1 - 2\pi m)\, 2\pi\delta(0) = [2\pi\delta(0)] \sum_{m=-\infty}^{\infty} 2\pi\delta(\omega T_1 - 2\pi m)$$

so that

$$\left[\sum_{m=-\infty}^{\infty} 2\pi\delta(\omega T_1 - 2\pi m) \right]^2 = [2\pi\delta(0)] \sum_{m=-\infty}^{\infty} 2\pi\delta(\omega T_1 - 2\pi m) \quad . \tag{33.8}$$

The object $\delta(0)$ is formally undefined, but in Appendix A we ascribe the meaning that $2\pi\delta(0) = 2N+1$ in the limit that $N \to \infty$ and we can always "undo the limit" when necessary. We shall firm up this idea in section (b) directly below. So we have shown then that

$$| Y''(z) |^2 = (1/T_1)^2 |Y'(\omega)|^2 = [2\pi\delta(0)] \sum_{m=-\infty}^{\infty} 2\pi\delta(\omega T_1 - 2\pi m) \tag{33.9}$$

or

$$\frac{|Y''(z)|^2}{[2\pi\delta(0)]} = (1/T_1)^2 \frac{|Y'(\omega)|^2}{[2\pi\delta(0)]} = \sum_{m=-\infty}^{\infty} 2\pi\delta(\omega T_1 - 2\pi m) \ . \tag{33.10}$$

Then from (33.6)

$$\frac{|X(\omega)|^2}{[2\pi\delta(0)]} = |X_{\mathbf{pulse}}(\omega)|^2 \frac{|Y''(z)|^2}{[2\pi\delta(0)]} = |X_{\mathbf{pulse}}(\omega)|^2 (1/T_1)^2 \frac{|Y'(\omega)|^2}{[2\pi\delta(0)]}$$

$$= |X_{\mathbf{pulse}}(\omega)|^2 \sum_{m=-\infty}^{\infty} 2\pi\delta(\omega T_1 - 2\pi m) \quad . \tag{33.11}$$

We shall now repeat the above set of steps for a finite pulse train.

(b) Finite Simple Pulse Train

Our finite pulse train always has pulses ranging from n = -N to N instead of from n = -∞ to ∞. We start off exactly as in the previous section but with limited sums,

$$x(t) = \sum_{n=-N}^{N} y_n\, x_{\mathbf{pulse}}(t - t_n) = \sum_{n=-N}^{N} x_{\mathbf{pulse}}(t - t_n) \tag{33.12}$$

$$X(\omega) = (1/T_1) X_{\mathbf{pulse}}(\omega)\, Y'(\omega) = X_{\mathbf{pulse}}(\omega)\, Y''(z) \tag{33.13}$$

$$Y''(z) = Y'(\omega)/T_1 = \sum_{n=-N}^{N} y_n\, e^{-i\omega n T_1} = \sum_{n=-N}^{N} e^{-i\omega n T_1} \quad . \tag{33.14}$$

In the last line we then use (13.3),

$$\sum_{n=-N}^{N} e^{ink} = 2\pi \left\{ \frac{\sin[(N+1/2)k]}{2\pi \sin(k/2)} \right\} \equiv 2\pi\, \delta_5(k,N) \qquad -\infty < k < \infty \qquad (13.3)$$

where δ_5 is a periodic delta function model discussed in Appendix A (b). Equation (33.14) becomes

$$Y''(z) = Y'(\omega)/T_1 = \sum_{n=-N}^{N} e^{-i\omega n T_1} = 2\pi\, \delta_5(\omega T_1, N) \qquad (33.15)$$

and then equation (33.13) says

$$X(\omega) = X_{pulse}(\omega)\, 2\pi\, \delta_5(\omega T_1, N)\ . \qquad (33.16)$$

This δ_5 is periodic with period 2π and has identical peaks separated by 2π. For finite N, these are peaks of finite width and height. Squaring, we find

$$|\, X(\omega)\,|^2 = |\, X_{pulse}(\omega)\,|^2\ [2\pi\, \delta_5(\omega T_1, N)]^2 \quad . \qquad (33.17)$$

Recalling the definition of the δ_6 delta function model from Appendix A (A.20),

$$2\pi\, \delta_6(k,N) \equiv \frac{[2\pi \delta_5(k,N)]^2}{(2N+1)} \qquad (A.20)$$

we obtain

$$\frac{|X(\omega)|^2}{(2N+1)} = |\, X_{pulse}(\omega)\,|^2\ \ 2\pi\, \delta_6(\omega T_1, N) \qquad (33.18)$$

and this is the finite pulse train result. We can then take the limit $N \to \infty$ and make use of (A.21),

$$\lim_{N\to\infty} \delta_6(\omega T_1, N) = \sum_{m=-\infty}^{\infty} \delta(\omega T_1 - 2\pi m) \qquad (A.21)$$

to find that

$$\lim_{N\to\infty} \left[\frac{|X(\omega)|^2}{(2N+1)}\right] = |\, X_{pulse}(\omega)\,|^2 \sum_{m=-\infty}^{\infty} 2\pi\, \delta(\omega T_1 - 2\pi m) \qquad (33.19)$$

and this replicates (33.11) with the promised connection $2\pi\delta(0) = \lim_{N\to\infty} (2N+1)$.

It is useful now to provide some side-by-side comparisons of results :

$$Y''(z) = Y'(\omega)/T_1 = \sum_{n=-\infty}^{\infty} e^{-i\omega n T_1} = \sum_{m=-\infty}^{\infty} 2\pi\,\delta(\omega T_1 - 2\pi m) \qquad \text{infinite} \qquad (33.4)$$

$$Y''(z) = Y'(\omega)/T_1 = \sum_{n=-N}^{N} e^{-i\omega n T_1} = 2\pi\,\delta_5(\omega T_1, N) \qquad \text{finite} \qquad (33.15)$$

$$X(\omega) = X_{pulse}(\omega) \sum_{m=-\infty}^{\infty} 2\pi\,\delta(\omega T_1 - 2\pi m) \qquad \text{infinite} \qquad (33.5)$$

$$X(\omega) = X_{pulse}(\omega)\; 2\pi\,\delta_5(\omega T_1, N)\,. \qquad \text{finite} \qquad (33.16)$$

$$\frac{|X(\omega)|^2}{[2\pi\delta(0)]} = |X_{pulse}(\omega)|^2 \sum_{m=-\infty}^{\infty} 2\pi\,\delta(\omega T_1 - 2\pi m) \qquad \text{infinite} \qquad (33.11)$$

$$\frac{|X(\omega)|^2}{(2N+1)} = |\,X_{pulse}(\omega)\,|^2\;\; 2\pi\,\delta_6(\omega T_1, N) \qquad \text{finite} \qquad (33.18)$$

One can interpret $\frac{|X(\omega)|^2}{[2\pi\delta(0)]}$ as the value of $|X(\omega)|^2$ *per pulse* in an infinite pulse train.

Things are OK again until we get to Section 34 (b) so I will repair it below:

(b) Spectral power density for a General Pulse Train

In Section 33 (d) we dealt with simple pulse trains. Here we consider the more general amplitude modulated pulse train. In all equations, one can replace $\sum_{n=-\infty}^{\infty}$ by $\sum_{n=-N}^{N}$ to adapt the equation to a finite pulse train instead of an infinite one. We start then with

$$x(t) = \sum_{n=-\infty}^{\infty} y_n\; x_{pulse}(t - t_n) \qquad (34.5)$$

$$X(\omega) = (1/T_1) X_{pulse}(\omega)\; Y'(\omega) = X_{pulse}(\omega)\; Y''(z) \qquad (34.6)$$

$$Y''(z) = Y'(\omega)/T_1 = \sum_{n=-\infty}^{\infty} y_n\, e^{-i\omega n T_1}\;. \qquad (34.7)$$

To find the frequency-domain power spectrum of a signal x(t), our first task is to compute $|\,X(\omega)\,|^2$. From (34.6) we get

$$|X(\omega)|^2 \;=\; |X_{\mathbf{pulse}}(\omega)|^2\,(1/T_1)^2\,|Y'(\omega)|^2 \;=\; |X_{\mathbf{pulse}}(\omega)|^2\,|\,Y''(z)\,|^2 \qquad z = e^{i\omega T_1} \tag{34.8}$$

We therefore must deal with the following object, using (34.7),

$$|\,Y''(z)\,|^2 \;=\; (1/T_1)^2\,|Y'(\omega)|^2 \;=\; \Big|\sum_{n=-\infty}^{\infty} y_n\, e^{-i\omega n T_1}\Big|^2$$

$$= \sum_{n=-\infty}^{\infty} y_n\, e^{-i\omega n T_1} \sum_{m=-\infty}^{\infty} y_m^*\, e^{+i\omega m T_1} \;=\; \sum_{n=-\infty}^{\infty}\sum_{m=-\infty}^{\infty} y_m^*\, y_n\, e^{i\omega(m-n)T_1} \tag{34.9}$$

so that

$$|\,X(\omega)\,|^2 \;=\; |\,X_{\mathbf{pulse}}(\omega)\,|^2 \sum_{n=-\infty}^{\infty}\sum_{m=-\infty}^{\infty} y_m^*\, y_n\, e^{i\omega(m-n)T_1} \qquad . \tag{34.10}$$

From box (34.4) we then have

$$\mathcal{E}(\omega) \equiv |X(\omega)|^2/2\pi \;=\; (1/2\pi)\,|\,X_{\mathbf{pulse}}(\omega)\,|^2 \sum_{n=-\infty}^{\infty}\sum_{m=-\infty}^{\infty} y_m^*\, y_n\, e^{i\omega(m-n)T_1} \tag{34.11}$$

$$\mathcal{P}(\omega) \;\equiv \frac{\mathcal{E}(\omega)}{T} \;= (1/2\pi T)\,|\,X_{\mathbf{pulse}}(\omega)\,|^2 \sum_{n=-\infty}^{\infty}\sum_{m=-\infty}^{\infty} y_m^*\, y_n\, e^{i\omega(m-n)T_1} \tag{34.12}$$

which we can write as (again using box (34.4) results)

$$\mathcal{E}(\omega) = \; T_1\,\mathcal{P}_{\mathbf{pulse}}(\omega) \sum_{n=-\infty}^{\infty}\sum_{m=-\infty}^{\infty} y_m^*\, y_n\, e^{i\omega(m-n)T_1} \tag{34.13}$$

$$\mathcal{P}(\omega) \;= T_1\,\mathcal{P}_{\mathbf{pulse}}(\omega)\,(1/T) \sum_{n=-\infty}^{\infty}\sum_{m=-\infty}^{\infty} y_m^*\, y_n\, e^{i\omega(m-n)T_1} \tag{34.14}$$

$$P \;=\; \int_{-\infty}^{\infty} d\omega\; \mathcal{P}(\omega) \quad . \qquad \textit{(33.31)} \tag{34.15}$$

Not knowing details of the y_n there is not much else we can do in these expressions.

(c) Pulse Trains with Repeated Sequences

Consider a pulse train composed of some general pulse shape $x_{\mathbf{pulse}}(t)$ whose amplitudes are repeated sequences of A,B. We shall compute the spectrum $X(\omega)$ and spectral power density $\mathcal{P}(\omega)$ by two different methods.

The first method is more or less by brute force, and it reveals a potential pitfall in using the $\delta(0)$ notation and shows a clean way to avoid the pitfall.

The second method, much simpler, is to use the Fourier Series results in box (15.12) applied to the repeating sequence.

We then state X(ω) and $\mathscr{P}(\omega)$ for a few special cases including various square waves.
At the very end we generalize the results to any repeated sequence A,B,C... .

Method 1: Brute Force Approach

Our starting point is (34.8) with (34.7), where we assume N is large and later we will take N→∞ :

$$X(\omega) = (1/T_1)X_{\mathbf{pulse}}(\omega)\, Y'(\omega) = X_{\mathbf{pulse}}(\omega)\, Y''(z) \qquad (34.6)$$

$$|X(\omega)|^2 = |X_{\mathbf{pulse}}(\omega)|^2\, (1/T_1)^2\, |Y'(\omega)|^2 = |X_{\mathbf{pulse}}(\omega)|^2\, |\, Y''(z)\, |^2 \qquad z = e^{i\omega T_1} \qquad (34.8)$$

$$Y''(z) = Y'(\omega)/T_1 = \sum_{n=-N}^{N} y_n\, e^{-i\omega n T_1}. \qquad (34.7)$$

The main problem is to compute Y''(z) and then square it. We have

$$\sum_{n=-N}^{N} y_n\, e^{-i\omega n T_1} = A \sum_{n\ \text{even}} e^{-i\omega n T_1} + B \sum_{n\ \text{odd}} e^{-i\omega n T_1} .$$

Now process the sums as follows, where

$$\sum_{n\ \text{even}} e^{-i\omega n T_1} = \sum_{m=-N/2}^{N/2} e^{-i\omega (2m) T_1} \quad \text{where we used } n = 2m$$

$$\sum_{n\ \text{odd}} e^{-i\omega n T_1} = \sum_{n=(-N-1)/2}^{(N-1)/2} e^{-i\omega (2m+1) T_1} \quad \text{where we used } n = 2m + 1 .$$

We assume N is very large, so we regard (N±1)/2 ≈ N/2 . We then find

$$Y''(z) = Y'(\omega)/T_1 = \sum_{n=-N}^{N} y_n\, e^{-i\omega n T_1} = [\, A + B\, e^{-i\omega T_1}] \sum_{m=-N/2}^{N/2} e^{-i\omega (2m) T_1} .$$

We now use (13.3),

$$\sum_{n=-N}^{N} e^{\mathbf{i}nk} = 2\pi \left\{ \frac{\sin[(N+1/2)k]}{2\pi \sin(k/2)} \right\} \equiv 2\pi\, \delta_5(k,N) , \qquad -\infty < k < \infty \qquad (13.3)$$

to write

$$\sum_{m=-N/2}^{N/2} e^{-i\omega (2m) T_1} = 2\pi\, \delta_5(2\omega T_1, N/2)$$

where δ_5 and δ_6 to come are explained in Appendix A (b). Therefore,

$$Y''(z) = [\, A + B\, e^{-i\omega T_1}]\, 2\pi\delta_5(2\omega T_1, N/2) \quad . \tag{34.16}$$

Using this Appendix A result,

$$\lim_{N\to\infty} \delta_5(k,N) = \sum_{m=-\infty}^{\infty} \delta(k-2\pi m) \tag{A.19}$$

we obtain the $N\to\infty$ limit for our spectrum

$$\begin{aligned} Y''(z) &= [\, A + B\, e^{-i\omega T_1}]\, 2\pi \sum_{m=-\infty}^{\infty} \delta(2\omega T_1 - 2\pi m) \\ &= (1/2)[\, A + B\, e^{-i\omega T_1}]\, (1/T_1)\; 2\pi \sum_{m=-\infty}^{\infty} \delta(\omega - m\omega_1/2) \\ &= (1/2)\, \omega_1 \sum_{m=-\infty}^{\infty} [\, A + B\,(-1)^m]\; \delta(\omega - m\omega_1/2) \end{aligned} \tag{34.17}$$

and correspondingly

$$X(\omega) = X_{pulse}(\omega)\, (1/2)\, \omega_1 \sum_{m=-\infty}^{\infty} [\, A + B(-1)^m\,]\, \delta(\omega - m\omega_1/2)\,. \tag{34.18}$$

There is a certain logic to the $[\, A + B\, e^{-i\omega T_1}]$ factor. If we set A = K and B = 0 we get one result, and if we set A = 0 and B = K we get the same result multiplied by $e^{-i\omega n T_1}$. The second pulse train is just the first pulse train shifted T_1 units to the right, and this adds phase $e^{-i\omega n T_1}$ as in (12.1).

If we were to square (34.18) and use our usual $2\pi\delta(0) = 2N+1$ association, we get a result that is off by a factor of 2. The reason is that our pre-limit sums are going from -N/2 to N/2, so we would get the right answer if we were to adjust and say $2\pi\delta(0) = N+1$. Rather than make an arm-waving argument to this effect, it is safer to continue along with our pre-limit expressions, having paused to take the limit for the spectrum X(ω) as in (34.18).

So, backing off again from limit, we square (34.16) to get

$$|Y''(z)|^2 = |A + Be^{-i\omega T_1}|^2\, [2\pi\, \delta_5(2\omega T_1, N/2)]^2\,.$$

Then from (A.20) applied with $N \to N/2$

$$\delta_6(k,N/2) \equiv \frac{1}{2\pi}\; \frac{[2\pi\delta_5(k,N/2)]^2}{(N+1)} \tag{A.20}$$

we get

$$|Y''(z)|^2 = |A + Be^{-i\omega T_1}|^2 \, [2\pi \, \delta_5(2\omega T_1, N/2)]^2$$

or

$$\frac{|Y''(z)|^2}{2\pi(N+1)} = |A + Be^{-i\omega T_1}|^2 \, \{ \frac{1}{2\pi} \, \frac{[2\pi\delta_5(2\omega T_1, N/2)]^2}{(N+1)} \} \; = \; |A + Be^{-i\omega T_1}|^2 \; \delta_6(2\omega T_1, N/2) \, .$$

Now for large N we ignore the difference between N and N + 1 and so on, so we divide both sides by 2 to get,

$$\frac{|Y''(z)|^2}{2\pi(2N+1)} \; = \; (1/2) \, |A + Be^{-i\omega T_1}|^2 \; \delta_6(2\omega T_1, N/2)$$

Notice that a very important factor of 1/2 appears on the right in the last step. We now insert the squared pulse spectrum to get

$$\frac{|X(\omega)|^2}{2\pi(2N+1)} \; = \; |X_{pulse}(\omega)|^2 \, \frac{|Y''(z)|^2}{2\pi(2N+1)} \; = \; |X_{pulse}(\omega)|^2 \; (1/2)|A + Be^{-i\omega T_1}|^2 \; \delta_6(2\omega T_1, N/2) \quad .$$

If we divide both sides by T_1 the left side is $\frac{|X(\omega)|^2}{2\pi T}$ where T is the length of the pulse train and this in turn equals $\mathcal{P}(\omega)$, all as shown in box (34.4). So for large N we have shown that

$$\mathcal{P}(\omega) \; = \; |X_{pulse}(\omega)|^2 \, (1/T_1)(1/2) \, |A + Be^{-i\omega T_1}|^2 \; \delta_6(2\omega T_1, N/2)$$

$$= \; \mathcal{P}_{pulse}(\omega)(1/2) \, |A + Be^{-i\omega T_1}|^2 \; 2\pi \, \delta_6(2\omega T_1, N/2) \; . \qquad (34.19)$$

Now at last we take the limit $N \to \infty$ and use

$$\lim_{N\to\infty} \delta_6(k,N) \; = \; \sum_{m=-\infty}^{\infty} \delta(k - 2\pi m) \qquad (A.21)$$

to get our desired infinite pulse train result

$$\mathcal{P}(\omega) \; = \; \mathcal{P}_{pulse}(\omega)(1/2) \, |A + Be^{-i\omega T_1}|^2 \; \sum_{m=-\infty}^{\infty} 2\pi \, \delta(2\omega T_1 \; - 2\pi m)$$

$$= \; \mathcal{P}_{pulse}(\omega)(1/4) \, |A + Be^{-i\omega T_1}|^2 \, (1/T_1) \; \sum_{m=-\infty}^{\infty} 2\pi \, \delta(\omega \; - m\omega_1/2)$$

$$= \; \mathcal{P}_{pulse}(\omega)(1/4) \, (1/T_1) \; \sum_{m=-\infty}^{\infty} |A + B(-1)^m \, |^2 \; 2\pi \, \delta(\omega \; - m\omega_1/2)$$

$$= \mathcal{P}_{\mathbf{pulse}}(\omega)(1/4)\,\omega_1 \sum_{m=-\infty}^{\infty} \{\, |A|^2 + |B|^2 + 2\mathrm{Re}(AB)(-1)^m \,\}\, \delta(\omega - m\omega_1/2)\,. \qquad (34.20)$$

Summarizing the key results:

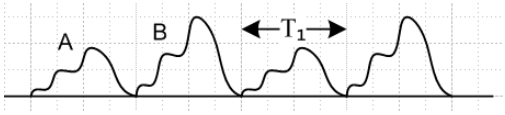


Fig 34.1

$$X(\omega) = X_{\mathbf{pulse}}(\omega)\,(1/2)\,\omega_1 \sum_{m=-\infty}^{\infty} [\, A + B(-1)^m \,]\, \delta(\omega - m\omega_1/2)\,. \qquad \textit{(34.18)}$$

$$\mathcal{P}(\omega) = \mathcal{P}_{\mathbf{pulse}}(\omega)\,(1/4)\,\omega_1 \sum_{m=-\infty}^{\infty} \{\, |A|^2 + |B|^2 + 2\mathrm{Re}(AB)(-1)^m \,\}\, \delta(\omega - m\omega_1/2) \qquad \textit{(34.20)}$$

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