Paradox on computing FT of 1/t PhL 4.5.13

This stuff now appears in Appendix C (c) and summary (g). I did get a verification of the sgn formula as shown below. The "paradox" is that it depends whether you use regular or generalized FT.

I am unable to compute the Fourier Transform of x(t) = 1/t

Method 1

Start off with

$$X(\omega) = \int_{-\infty}^{\infty} dt\, x(t)\, e^{-i\omega t} \qquad \text{projection = transform} \qquad (1.1)$$

Try this for x(t) = 1/t. In the **generalized FT**, I would run the contour below the pole.

For $\omega > 0$, we can close down because $-i\omega t = -i\omega(-i\infty) = -\omega\infty$. There is no pole, so get 0.
For $\omega < 0$ we can close up, we get the pole CCW so get $2\pi i$. The result is then

$$X(\omega) = 2\pi i\, \theta(-\omega)$$

This is like the FT of the Heaviside step situation.

Method 2

Use the actual FT, so we have

$$X(\omega) = \int_{-\infty}^{\infty} dt\, (1/t)\, e^{-i\omega t} \qquad \text{projection = transform} \qquad (1.1)$$

This is undefined, so we try if using

$$\frac{1}{t \mp i\varepsilon} = pf(1/t) \pm i\pi\delta(t)$$

Then we find

$$X(\omega) = \int_{-\infty}^{\infty} dt \frac{1}{t \mp i\varepsilon}\, e^{-i\omega t} = \fint_{-\infty}^{\infty} dt\ (1/t)\, e^{-i\omega t} \pm \int_{-\infty}^{\infty} dt\, i\pi\delta(t)\, e^{-i\omega t}$$

Then the integral is easy to do

$$\fint_{-\infty}^{\infty} dt\ (1/t)\, [\, \cos(\omega t) - i\sin(\omega t)\,] = -i \fint_{-\infty}^{\infty} dt\ (1/t)\, \sin(\omega t)$$

$= -2i \int_0^\infty dt\ (1/t)\sin(\omega t) = -2i\ (\pi/2)\ \text{signum}(\omega)$

$= -i\pi\ \text{signum}(\omega)$ ω is real

Then we get

$X(\omega) = -i\pi\ \text{signum}(\omega) \pm i\pi$

This result makes no sense at all to me, I don't even know what it means.

Method 3

If I just assume the integral goes right through the pole I get

$X(\omega) = \oint_{-\infty}^{\infty} dt\ (1/t)\ e^{-i\omega t} = -i\pi\ \text{signum}(\omega)$

Then the Fourier Transform of $1/(\pi t)$ is given by

$FT\ [1/(\pi t)] = -i\pi\ \text{signum}(\omega)$

This seems to agree with my PDF source

The signal $1/(\pi t)$ has Fourier transform

$$-\mathrm{j}\ \mathrm{sgn}(f) = \begin{cases} -\mathrm{j}, & \text{if } f > 0 \\ 0, & \text{if } f = 0 \\ \mathrm{j}, & \text{if } f < 0 \end{cases}$$