29. Examples of Image Spectra

Example 1

Consider the following finite length digital signal, which we assume has time spacing T_1,

y_n (n=-5..5) = { 1,2,3,3,4,1,0.2,-1,-2,-2,-1}

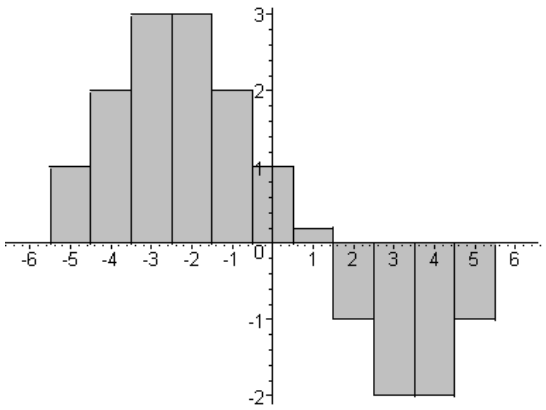


Fig 29.1

All samples other than those shown are 0. Using an arbitrary pulse x_{pulse}, we can construct an amplitude-modulated pulse train w(t) whose spectrum is W(ω) as shown in summary box (25.4),

$$w(t) = \sum_{n=-\infty}^{\infty} y_n \, x_{pulse}(t - t_n) \tag{29.1}$$

$$W(\omega) = (1/T_1) X_{pulse}(\omega) \, Y'(\omega) , \tag{29.2}$$

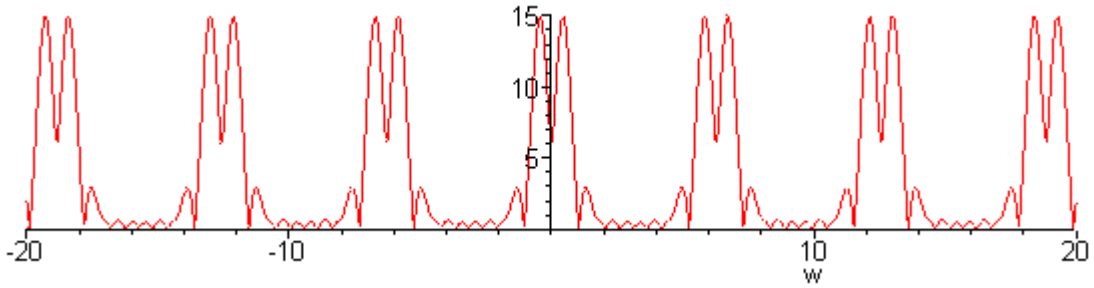
where from summary box (23.5),

$$Y'(\omega) \equiv T_1 \sum_{n=-\infty}^{\infty} y_n \, e^{-i\omega n T_1} \quad . \tag{29.3}$$

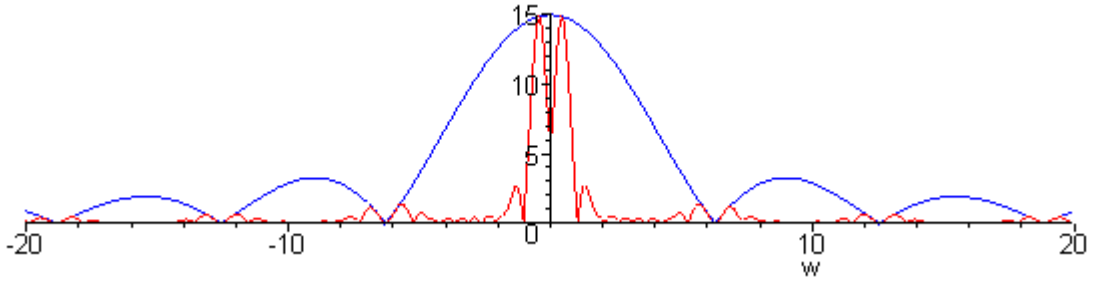
If we were to select $x_{pulse}(t)$ to be a box of height 1 and width T_1, then we could regard the stair-step outline function shown in Fig 29.1 as a candidate analog signal y(t) whose samples are the y_n. In this special case, y(t) = w(t) so Y(ω) = W(ω). From (9.2) we have $X_{box}(\omega) = T_1 \mathrm{sinc}(\omega T_1/2)$ so that using (29.2),

$$Y(\omega) = W(\omega) = \mathrm{sinc}(\omega T_1/2) \, Y'(\omega) = T_1 \, \mathrm{sinc}(\omega T_1/2) \sum_{n=-\infty}^{\infty} y_n \, e^{-i\omega n T_1}$$

Y(ω) is the Fourier Integral Transform spectrum of the stair-step signal in Fig 29.1. Here are some plots



Plot of |Y'(ω)|

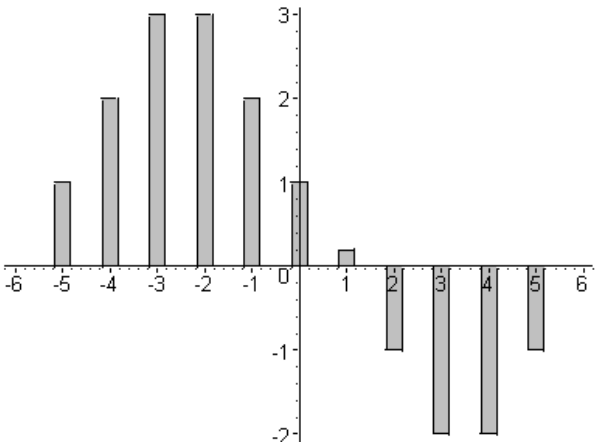


Plot of |W(ω)| in red, 15* |sinc(ωT₁/2)| in blue

In this example, we see the expected image spectra in Y'(ω) in the first plot, but these spectra are not present in the spectrum W(ω) of the physical signal w(t).

Example 2

We use the same sample values y_n as in the previous Example, but the box width is $\tau = T_1/3$ instead of $\tau = T_1$. So our analog signal w(t) is now this set of spikes,

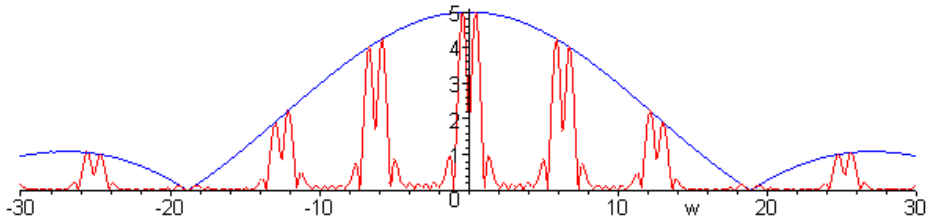


Signal w(t) for Example 2.

From (9.2) now have $X_{box}(\omega) = (T_1/3)\ \mathrm{sinc}(\omega * T_1/6)$, so the spectrum of this w(t) is

$W(\omega) = (1/T_1)[\ (T_1/3)\ \mathrm{sinc}(\omega * T_1/6)]Y'(\omega) = (1/3)\ \mathrm{sinc}(\omega * T_1/6)Y'(\omega)$

with the same Y'(ω) as in the previous example. Here is a plot of the new W(ω),



Plot of $|W(\omega)|$ for Example 2 in red, $5\ |\mathrm{sinc}(\omega * T_1/6)|$ in blue.

In this Example, the physical analog signal w(t) *does* have some strong image spectra in addition to the central main spectrum

Suppose for the corresponding analog function y(t) we take the stair-step outline of the digital signal.

$$w(t) = y(t)\ x(t) = \sum_{n=-\infty}^{\infty} y_n\ x_{\mathbf{pulse}}(t - t_n)$$

where

$x_{\mathbf{pulse}}(t) = 1\ [\ \theta(t + T_1/2) - \theta(t - T_1/2)\ ]$ = box of height 1 and width T_1