This is the Title PhL 3.26.05

This is just a temp working file, everything here has been taken care of.

How would section 20 look for a finite pulse train?

$$x(t) = \sum_{n=-N}^{N} x_{pulse}(t - t_n) \qquad t_n = nT_1 \qquad \text{pulse train} \qquad (14.17)$$

$$X(\omega) = X_{pulse}(\omega)\, 2\pi\delta_5(\omega T_1, N) = c(\omega)\, 2\pi\, T_1\delta_5(\omega T_1, N) \qquad (14.19)$$

$$x_{pulse}(t) = T_1\ \delta(t) = \delta(t/T_1) \qquad \text{// dimensionless}$$

$$X_{pulse}(\omega) = T_1 \qquad \textit{(8.3)}$$

$$c(\omega) = 1 \qquad \textit{(14.12) item 2}$$

$$x(t) = \sum_{n=-N}^{N} \delta(t - nT_1) \qquad \textit{(14.12) item 1} \qquad (20.1)$$

$$X(\omega) = 2\pi\, T_1\delta_5(\omega T_1, N) \qquad \textit{(14.19)}$$

$$w(t) = y(t)\, x(t) = \sum_{n=-N}^{N} y(t)\delta(t - nT_1) = \sum_{n=-N}^{N} y(t_n)\delta(t - nT_1) = \sum_{n=-N}^{N} y_n\delta(t - nT_1) \qquad (20.3)$$

Equation (20.5) no longer works without real delta functions!!! So we no longer have

$$W(\omega) = Y(\omega) + \sum_{m \neq 0} Y(\omega - m\omega_1) \quad .\ \text{not true for finite train} \qquad (20.7)$$

So I really don't want to "add" anything to Section 20. Maybe the discrete FT can help here???

I see no further changes needed before Section 25.

Section 25: I see no need to add anything here!

So I think now I can jump to Section 32 and start worrying about things there.

Section 32 is OK for any x(t) really, no need to add anything about finite pulse trains here.

Section 33 definitely needs some finite pulse train addition!

$$x(t) = \sum_{n=-N}^{N} x_{\mathbf{pulse}}(t - nT_1) \qquad \textit{(14.17)} \qquad (33.xx)$$

$$X(\omega) = X_{\mathbf{pulse}}(\omega)\, 2\pi\delta_5(\omega T_1, N) \qquad \textit{(14.10)} \qquad (33.xx)$$

$$| X(\omega) |^2 = | X_{\mathbf{pulse}}(\omega)|^2 \; [2\pi\delta_5(\omega T_1, N)]^2$$

From (A.21) we now replace

$$[2\pi\delta_5(\omega T_1, N)]^2 = (2N+1)\, 2\pi\, \delta_6(k, N)$$

so that

$$| X(\omega) |^2 = (2N+1) \, | X_{\mathbf{pulse}}(\omega)|^2 \; 2\pi\, \delta_6(k, N)$$

$$\mathcal{P}(\omega) = \frac{|X(\omega)|^2}{2\pi} = (2N+1) \, \frac{|X_{\mathbf{pulse}}(\omega)|^2}{2\pi} \; 2\pi\, \delta_6(k, N)$$

$$\mathcal{P}_1(\omega) = \frac{|X(\omega)|^2}{2\pi} = \frac{|X_{\mathbf{pulse}}(\omega)|^2}{2\pi} \; 2\pi\, \delta_6(k, N)$$

Then we sort of stop for a while with this result which is exact for finite N.

Section 34 what happens here? For a finite train we get down to here with no changes

$$\langle r_{\mathbf{x}}(t) \rangle = \sum_{n=-N}^{N} \sum_{n=-N}^{N} \langle a_m a_n \rangle \, r_{\mathbf{pulse}}[t + (m-n)T_1)] \qquad (34.7)$$

$$\langle |X(\omega)|^2 \rangle = |X_{\mathbf{pulse}}(\omega)|^2 \sum_{n=-N}^{N} \sum_{m=-N}^{N} \langle a_m a_n \rangle \, e^{i\omega(m-n)T_1} \qquad (34.9)$$

No changes at all here, just show finite sums is all! I added a trailer sectino showing the key results with finite sums.

Section 35 what happens here? Now we have to do a separate finite N section.

Finite Pulse train with Random Coefficients

Using the above forms for the coefficient averages, (34.9) becomes

$$\langle |X(\omega)|^2 \rangle = |X_{\mathbf{pulse}}(\omega)|^2 \; \{ \; \alpha \sum_{n=-N}^{N} \sum_{m \neq n} [\, e^{i\omega(m-n)T_1}] \; + \beta \sum_{n=-N}^{N} \sum_{m=n} [1] \; \} \qquad (35.4)$$

To evaluate the first double sum, we write it as

$$\sum_{n=-N}^{N} \sum_{m \neq n} [\, e^{i\omega(m-n)T_1}] = \sum_{n=-N}^{N} \{ \sum_{m=-N}^{N} [\, e^{i\omega(m-n)T_1}] - 1 \}$$

$$= (\sum_{n=-N}^{N} e^{+i\omega n T_1}) (\sum_{m=-N}^{N} e^{-i\omega m T_1}) - \sum_{n=-\infty}^{\infty} 1 = | \sum_{n=-N}^{N} e^{+i\omega n T_1} |^2 - \sum_{n=-N}^{N} 1$$

We now quote two results from Appendix A

$$\sum_{n=-N}^{N} e^{ink} = 2\pi \{ \frac{\sin[(N+1/2)k]}{2\pi \sin(k/2)} \} = 2\pi\delta_5(k,N)$$

$$\sum_{n=-N}^{N} 1 = (2N+1) \qquad \textit{(A.32)}$$

so the first double sum in (35.4) becomes

$$\sum_{n=-N}^{N} \sum_{m \neq n} [\, e^{i\omega(m-n)T_1}] = [2\pi\delta_5(\omega T_1,N)]^2 - (2N+1) \qquad (35.5)$$

The second double sum in (35.4) is just

$$\sum_{n=-N}^{N} \sum_{m=n} [1] = \sum_{n=-N}^{N} [1] = (2N+1) \qquad (35.6)$$

and therefore we may write (35.4) as

$$\langle |X(\omega)|^2 \rangle = |X_{pulse}(\omega)|^2 \{ \alpha \, [\, [2\pi\delta_5(\omega T_1,N)]^2 - (2N+1) \,] + \beta \, (2N+1) \}$$

or

$$\frac{\langle |X(\omega)|^2 \rangle}{(2N+1)} = |X_{pulse}(\omega)|^2 \{ \alpha \, [\frac{[2\pi\delta_5(\omega T_1,N)]^2}{(2N+1)} - 1 \,] + \beta \} \qquad (35.7)$$

$$\frac{\langle |X(\omega)|^2 \rangle}{(2N+1)} = |X_{pulse}(\omega)|^2 \{ \alpha \, \frac{[2\pi\delta_5(\omega T_1,N)]^2}{(2N+1)} + (\beta-\alpha) \} \qquad (35.7)$$

$$\frac{\langle |X(\omega)|^2 \rangle}{(2N+1)} = |X_{pulse}(\omega)|^2 \{ (1/4) \, \frac{[2\pi\delta_5(\omega T_1,N)]^2}{(2N+1)} + (1/4) \} \qquad (35.7)$$

Now we divide both sides by 2π. The left side then becomes $\langle \mathcal{P}(\omega) \rangle / (2N+1) = \langle \mathcal{P}_1(\omega) \rangle$ as in (33.6). And $|X_{pulse}(\omega)|^2/2\pi = \mathcal{P}_{pulse}(\omega)$ so,

$$\langle \mathcal{P}_1(\omega) \rangle = \mathcal{P}_{pulse}(\omega) \{ \alpha \frac{[2\pi\delta_5(\omega T_1, N)]^2}{(2N+1)} + (\beta - \alpha) \} \quad (35.8)$$

At this point Appendix A says

$$\delta_6(k,N) \equiv \frac{1}{2\pi} \frac{[2\pi\delta_5(\omega T_1, N)]^2}{(2N+1)} = \frac{1}{2N+1} \frac{\sin^2[(N+1/2)k]}{2\pi \sin^2(k/2)} \quad (A.21)$$

so I then have

$$\frac{\langle |X(\omega)|^2 \rangle}{(2N+1)} = |X_{pulse}(\omega)|^2 \{ \alpha\, 2\pi\, \delta_6(k,N) + (\beta - \alpha) \} \quad (35.7)$$

Divide by 2π to get

$$\langle \mathcal{P}_1(\omega) \rangle = \mathcal{P}_{pulse}(\omega) \{ (\beta - \alpha) + \alpha\, 2\pi\, \delta_6(k,N) \}$$

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(a) Spectral Energy Density of an Finite Pulse Train

We start with

$$x(t) = \sum_{n=-\infty}^{\infty} x_{pulse}(t - nT_1) \quad (14.1) \quad (33.1)$$

$$X(\omega) = \sum_{m=-\infty}^{\infty} X_{pulse}(m\omega_1)\ 2\pi\ \delta(\omega T_1 - 2\pi m) \quad (14.4) \quad (33.2)$$