This is the Title PhL 3.26.05

This has been added as Appendix E to spectral doc.

Appendix E: Table of Transforms

Fourier Integral Transform: x(t) aperiodic and continuous, X(ω) continuous, no image spectra

$X(\omega) = \int_{-\infty}^{\infty} dt\, x(t)\, e^{-i\omega t}$ projection = transform *(1.1)*

$x(t) = (1/2\pi) \int_{-\infty}^{\infty} d\omega\, X(\omega)\, e^{+i\omega t}$ expansion = inverse transform *(1.2)*

Generalized Fourier Integral Transform: recovery contour passes below all singularities of X(ω)

$X(\omega) = \int_{0}^{\infty} dt\, x(t)\, e^{-i\omega t}$ projection = transform *(6.4)*

$x(t) = (1/2\pi) \int_{-ci-\infty}^{-ci+\infty} d\omega\, X(\omega)\, e^{+i\omega t}$ expansion = inverse transform *(6.5)*

Fourier Cosine Transform

$X_c(\omega) = 2 \int_{0}^{\infty} dt\, x(t) \cos(\omega t)$ projection = transform

$x(t) = (1/\pi) \int_{0}^{\infty} d\omega\, X_c(\omega) \cos(\omega t)$ expansion = inverse transform *(1.7)*

Fourier Sine Transform

$X_s(\omega) = 2 \int_{0}^{\infty} dt\, x(t) \sin(\omega t)$ projection = transform

$x(t) = (1/\pi) \int_{0}^{\infty} d\omega\, X_s(\omega) \sin(\omega t)$ expansion = inverse transform *(1.8)*

Fourier Integral Transform in Appendix C notation: k is an arbitrary convention constant

$\hat{x}(\omega) = k \int_{-\infty}^{\infty} dt\, x(t)\, e^{-i\omega t}$ projection = transform

$x(t) = (1/2\pi k) \int_{-\infty}^{\infty} d\omega\, \hat{x}(\omega)\, e^{+i\omega t}$ expansion = inverse transform

$\hat{f}(\omega) = k \int_{-\infty}^{\infty} du\, f(u)\, e^{-i\omega u}$ Fourier transform of f(u)

$f^{\wedge -1}(t) = (1/2\pi k) \int_{-\infty}^{\infty} du\ f(u)\ e^{+iut}$ inverse Fourier transform of f(u) *(C.1)*

Fourier Series Transform: x(t) periodic and continuous with period T_1, spectrum is discrete

$$x(t) = \sum_{n=-\infty}^{\infty} x_{pulse}(t - nT_1) \qquad (14.1)$$

Complex form:

$$c_m \equiv (1/T_1) \int_{-\infty}^{\infty} dt\ x_{pulse}(t)\ e^{-im\omega_1 t} = (1/T_1) \int_0^{T_1} dt\ x(t)\ e^{-im\omega_1 t} \qquad (14.16)$$

$$x(t) = \sum_{m=-\infty}^{\infty} c_m\ e^{+im\omega_1 t} \qquad (15.1)$$

$$c_m = c(m\omega_1) = (1/T_1)X_{pulse}(m\omega_1) \qquad (14.8) \text{ and } (14.10)$$

Real form:

$$a_m \equiv (2/T_1) \int_{-\infty}^{\infty} dt\ x_{pulse}(t) \cos(m\omega_1 t) = (2/T_1) \int_0^{T_1} dt\ x(t) \cos(m\omega_1 t) \qquad (15.6)$$

$$b_m \equiv (2/T_1) \int_{-\infty}^{\infty} dt\ x_{pulse}(t) \sin(m\omega_1 t) = (2/T_1) \int_0^{T_1} dt\ x(t) \sin(m\omega_1 t) \qquad (15.7)$$

$$x(t) = a_0/2 + \sum_{m=1}^{\infty} a_m \cos(m\omega_1 t) + \sum_{m=1}^{\infty} b_m \sin(m\omega_1 t) \qquad (15.9)$$

Laplace Transform: Right-sided (causal) x(t) vanishes for $t < 0$, x(t) and $\mathcal{X}(s)$ are continuous

$\mathcal{X}(s) = \int_0^{\infty} dt\ x(t)\ e^{-st}$ projection = transform *(6.9)*

$x(t) = (1/2\pi i) \int_{c-i\infty}^{c+i\infty} ds\ \mathcal{X}(s)\ e^{+is}$ expansion = inverse transform *(6.10)*

Relation to the Fourier Integral Transform:

$\mathcal{X}(s) = X(s/i)$ $X(\omega) = \mathcal{X}(i\omega)$ *(6.8)*

Digital Fourier Transform: $x(t_n)$ aperiodic, spectrum $X'(\omega)$ is continuous and contains image spectra

$$X'(\omega) \equiv \sum_{n=-\infty}^{\infty} \Delta t\ x(t_n)\ e^{-i\omega t_n} \qquad \text{projection = transform} \qquad (22.2)$$

$$x(t_n) = \frac{1}{2\pi} \int_{-\omega_1/2}^{\omega_1/2} d\omega\ X'(\omega)\ e^{+i\omega t_n} \qquad \text{expansion = inversion} \qquad (22.4)$$

or

$$X'(\omega) \equiv T_1 \sum_{n=-\infty}^{\infty} x_n e^{-i\omega n T_1} \qquad \text{projection = transform} \qquad x_n = x(t_n).$$

$$x_n = \frac{1}{2\pi} \int_{-\omega_1/2}^{\omega_1/2} d\omega\, X'(\omega)\, e^{+i\omega n T_1} \qquad \text{expansion = inversion}$$

where: $T_1 = \Delta t$ $\omega_1 = 2\pi/T_1 = 2\pi/\Delta t$ $t_n = n\,\Delta t = n\,T_1$.

Relation to the Fourier Integral Transform:

$$X'(\omega) = \sum_{m=-\infty}^{\infty} X(\omega - m\omega_1) = [\, X(\omega) + \sum_{m \neq 0} X(\omega - m\omega_1)\,] \qquad \text{// image spectra} \qquad \textit{(23.1)}$$

Z Transform: $x(t_n)$ is aperiodic, spectrum X"(z) is continuous and contains image spectra

$$X''(z) = \sum_{n=-\infty}^{\infty} x_n z^{-n} \qquad \text{projection = transform} \qquad \textit{(24.3)}$$

$$x_n = \frac{1}{2\pi i} \int_C dz\, X''(z)\, z^{n-1} \qquad \text{expansion = inversion} \qquad \textit{(24.4)}$$

where contour C goes once counterclockwise around the unit circle in the z-plane.

Relation to the Digital Fourier Transform and the Fourier Integral Transform:

$$X''(z) \equiv \frac{1}{T_1} X'(\omega) = \frac{1}{T_1} \sum_{m=-\infty}^{\infty} X(\omega - m\omega_1) \qquad z = e^{i\omega T_1}$$

Discrete Fourier Transform of a Pulse Train : $x(t_n)$ periodic, spectrum discrete, m integer

$$x(t_m) = \sum_{n=-\infty}^{\infty} x_{pulse}(t_m - nT_1) \qquad \text{sampled pulse train, } t_m = (m/N)T_1$$

$$c'_m \equiv (1/N) \sum_{n=-\infty}^{\infty} x_{pulse}(t_n)\, e^{-imn(2\pi/N)} \qquad \text{projection = transform} \qquad \textit{(27.9)}$$

$$x(t_n) = \sum_{m=0}^{N-1} c'_m\, e^{+imn(2\pi/N)} \qquad \text{expansion = inverse transform} \qquad \textit{(27.11), (27.12)}$$

Discrete Fourier Transform : A is an arbitrary convention constant

$$c'_m \equiv (A/N) \sum_{n=0}^{N-1} x_n e^{-imn(2\pi/N)} \qquad m = 0,1...N-1 \qquad \text{projection = transform}$$

$$x_n = (1/A) \sum_{m=0}^{N-1} c'_m e^{+imn(2\pi/N)} \qquad n = 0,1,...N-1 \qquad \text{expansion = inverse transform} \qquad \textit{(27.22)}$$

Hilbert Transform:

$$X_h(\omega) = (1/\pi) \int_{-\infty}^{\infty} d\omega' \frac{X(\omega')}{\omega-\omega'} \qquad \text{projection = transform}$$

$$X(\omega) = -(1/\pi) \int_{-\infty}^{\infty} d\omega' \frac{X_h(\omega')}{\omega-\omega'} \qquad \text{expansion = inverse transform} \qquad \textit{(C.41)}$$