Old Section 36 (c) PhL 3.19.13

I don't think this section has any use. I don't think the PSK remarks are correct since there you are really talking about a phase in the time domain, not the ω domain. I don't know that the significance of the phase of X(ω) is, so why try to compute its statistical average!

(c) derivation of <phase of X(ω)>

Finally, we might ask if there is something that can be said for the average phase of a statistical pulse train. We already have statements about the average full spectrum, and about the average magnitude squared. If we define the phase of X(ω) by $X(\omega)= |X(\omega)|e^{i\varphi}$ with $-\pi < \varphi \le \pi$, then it is easy to show from (36.2) that

$$\varphi^{i}(\omega) = \tan^{-1}\left\{ \frac{\sum_{n=-\infty}^{\infty} a^{i}_{n} \sin(n\omega T_1)}{\sum_{n=-\infty}^{\infty} a^{i}_{n} \cos(n\omega T_1)} \right\} \tag{36.8}$$

where we extend the definition of $\tan^{-1}$ to the range $(-\pi.\pi)$. So here is our not-very-satisfying result,

$$\langle\varphi(\omega)\rangle = (1/I) \sum_{i=1}^{I} \left[\tan^{-1}\left\{ \frac{\sum_{n=-\infty}^{\infty} a^{i}_{n} \sin(n\omega T_1)}{\sum_{n=-\infty}^{\infty} a^{i}_{n} \cos(n\omega T_1)} \right\} \right] \tag{36.9}$$

For random coefficients, one might expect $\langle\varphi(\omega)\rangle = 0$ since it is hard to imagine what else it could be. On the other hand, one could certainly find *seemingly* random coefficients for which this is not true.
For example, in N-ary phase shift keying, data words are encoded in bit patterns which cause the signal phase to jump between fixed points on a circular constellation in phase space. If a data word is repeated many times, one would expect <φ(ω)>

those that would be used to generate an N-ary phase-shift-keyed (N-PSK) signal for a constant , in which case the average phase should peak at N distinct values.