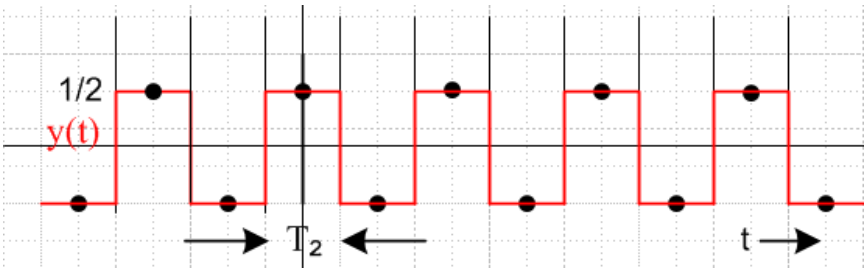
If p = 1, then AMI is just a square wave of amplitude V and period T_1. Finally after much mucking around, I show this in Section 25 : [the T_2 appearing here is half T_1]



$$X(\omega) = \sum_{m=1\,3\,5}^{\infty} \text{sinc}(m\frac{\pi}{2})\, 2\pi\delta(\omega - m\omega_2/2) = \sum_{m=1\,3\,5}^{\infty} \text{sinc}(m\frac{\pi}{2})\, 2\pi\delta(\omega - m\omega_1) \qquad (25.12)$$

I have to multiply by 2V to get the amplitude right, so

$$X(\omega) = 2V \sum_{m=1\,3\,5}^{\infty} \text{sinc}(m\frac{\pi}{2})\, 2\pi\delta(\omega - m\omega_1) = 2V\, T_1 \sum_{m=1\,3\,5}^{\infty} \text{sinc}(m\frac{\pi}{2})\, 2\pi\delta(\omega T_1 - m\omega_1/T_1)$$

$$= V\, T_1\, \Sigma_{m_odd}\, \text{sinc}(m\frac{\pi}{2})\, 2\pi\delta(\omega T_1 - m\omega_1/T_1)$$

$$= V\, T_1\, \Sigma_m\, \text{sinc}(m\frac{\pi}{2})\, 2\pi\delta(\omega T_1 - m\omega_1/T_1)$$

Now I can manually square

$$|X(\omega)|^2 = V^2T_1^2\, \Sigma_m\, \text{sinc}(m\frac{\pi}{2})\, 2\pi\, \delta(\omega T_1 - m\omega_1/T_1)\, \Sigma_{m'}\, \text{sinc}(m'\frac{\pi}{2})\, 2\pi\, \delta(\omega T_1 - m\omega_1/T_1)$$

$$= V^2T_1^2\, \Sigma_m\, \text{sinc}^2(m\frac{\pi}{2})\, 2\pi\, \delta(\omega T_1 - m\omega_1/T_1)\, 2\pi\delta(0)$$

so that

$$|X(\omega)|^2/\, [2\pi\delta(0)] = V^2T_1^2\, \Sigma_m\, \text{sinc}^2(m\frac{\pi}{2})\, 2\pi\, \delta(\omega T_1 - m\omega_1/T_1)$$

$$|X(\omega)|^2/\, [2\pi\delta(0)] = V^2T_1^2\, 2 \sum_{m=1\,3\,5}^{\infty} \text{sinc}^2(m\frac{\pi}{2})\, 2\pi\, \delta(\omega T_1 - m\omega_1/T_1)$$

$$|X(\omega)|^2/\,[2\pi\delta(0)] = V^2T_1\,2 \sum_{m=1\,3\,5}^{\infty} \mathrm{sinc}^2(m\frac{\pi}{2})\,2\pi\,\delta(\omega - m\omega_1)$$

I can express this in terms of T_2 as follows

$$|X(\omega)|^2/\,[2\pi\delta(0)] = V^2\,2T_2\,2 \sum_{m=1\,3\,5}^{\infty} \mathrm{sinc}^2(m\frac{\pi}{2})\,2\pi\delta(\omega - m\omega_2/2)$$

In my AMI world I have T_1 set to the pulse width, so my AMI limit I **expect** to be this

$$|X(\omega)|^2/\,[2\pi\delta(0)] = (2V)^2T_1 \sum_{m=1\,3\,5}^{\infty} \mathrm{sinc}^2(m\frac{\pi}{2})\,2\pi\delta(\omega - m\omega_1/2) \qquad **********$$

Meanwhile, here is my AMI result for general p

$$\frac{<|X(\omega)|^2>}{2\pi\delta(0)} = |X_{\mathbf{pulse}}(\omega)|^2 \;\frac{(1-a^2)\,[\sin^2(\omega T_1/2)]}{1+a^2-2a\cos(\omega T_1)} \qquad a \equiv (1-2p) \qquad \text{AMI}$$

$$X_{\mathbf{pulse}}(\omega) = (VT_1)\,\mathrm{sinc}(\omega T_1/2)$$

If I take p→ 1, then a = -1 and we have

$$\frac{<|X(\omega)|^2>}{2\pi\delta(0)} = |X_{\mathbf{pulse}}(\omega)|^2 \;\frac{(1-a^2)\,[\sin^2(\omega T_1/2)]}{1+1+2\cos(\omega T_1)}$$

$$= |X_{\mathbf{pulse}}(\omega)|^2 \;\frac{(1-a^2)\,[\sin^2(\omega T_1/2)]}{2+2\cos(\omega T_1)}$$

$$= |X_{\mathbf{pulse}}(\omega)|^2 \;\frac{(1-a^2)\,[\sin^2(\omega T_1/2)]}{2(1+\cos(\omega T_1))}$$

$$= |X_{\mathbf{pulse}}(\omega)|^2 \;\frac{(1-a^2)\,[\sin^2(\omega T_1/2)]}{4\cos^2(\omega T_1/2)})$$

$$= |X_{\mathbf{pulse}}(\omega)|^2 \quad (1/4)\,(1-a^2)\,\tan^2(\omega T_1/2)$$

$$= |X_{\mathbf{pulse}}(\omega)|^2 \quad (1/4)\,(1-a^2)\,\tan^2(\pi\frac{\omega}{\omega_1})$$

Footnote: I found a PDF with the spectrum stated this way

Evaluating on the unit circle we get the power spectrum

$$S_A(e^{j2\pi fT}) = 2p(1-p)\ \frac{1-\cos(2\pi fT)}{1+(1-2p)^2-2(1-2p)\cos(2\pi fT)}\ . \qquad (19.20)$$

If I take p = 1 in this result, the leading factor vanishes, the denominator is 1 + 1 + 2 cos(2πfT) so this result seems to have the same problem, it is no doubt the same result. The PDF plots the thing

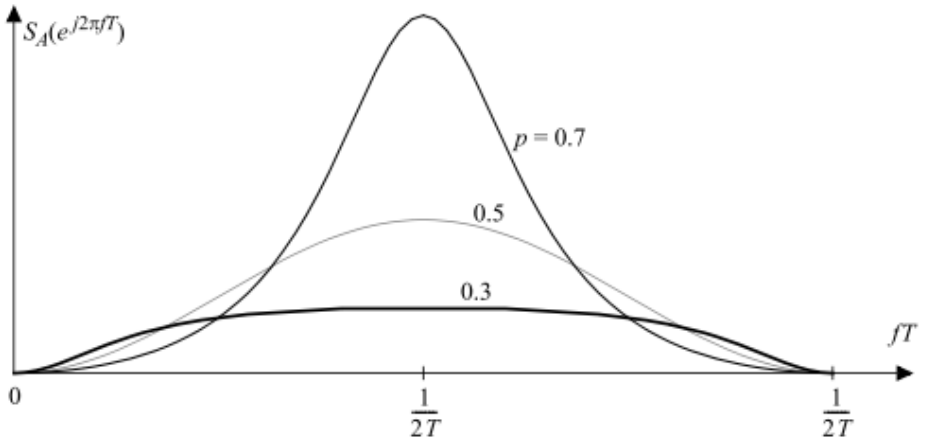


Fig. 19-7. Power spectrum for the AMI encoder output, neglecting the effect of the transmit pulse $g(t)$, for different density of "ones" in the information bit stream.

Go back now to my result

$$f(\omega) = |X_{pulse}(\omega)|^2 \quad (1/4)\,(1-a^2)\tan^2\left(\pi\frac{\omega}{\omega_1}\right)$$

The tangent blows up at $\pi\frac{\omega}{\omega_1} = \pi/2$. We can see f(ω) is periodic.

$$\pi\frac{\omega+\Delta\omega}{\omega_1} = \pi\frac{\omega}{\omega_1} + 2\pi$$

$$\frac{\omega+\Delta\omega}{\omega_1} = \frac{\omega}{\omega_1} + 2$$

$$\omega + \Delta\omega = \omega + 2\omega_1 \qquad \Delta\omega = 2\omega_1$$

So whatever happens repeats with period $\Delta\omega = 2\omega_1$. So we need only examine the region near π/2. Surely I can show a delta function there. So go back to

$$f(\omega) \equiv \frac{(1-a^2)\,[\sin^2(\omega T_1/2)]}{1+a^2-2a\cos(\omega T_1)}$$

Change variable to $x = \omega T_1/2$ so we have

$$f(\omega(x)) = \frac{(1-a^2)\,[\sin^2(x)]}{1+a^2-2a\cos(2x)}$$

Now let's look near the singular point

$$x = \pi/2 + \delta x$$

$$\sin(x) = \sin(\pi/2+\delta x) = \cos(\delta x) \approx 1$$

$$\cos(2x) = \cos(\pi + 2\delta x) = -\cos(2\delta x)$$

So near the problem we have

$$f(\omega(x)) = \frac{(1-a^2)}{1+a^2+2a\cos(2\delta x)}$$

Now expand

$$\cos(2\delta x) = 1 - (2\delta x)^2/2 = 1 - 2\,\delta x^2$$

Then denominator is

$$1 + a^2 + 2a(1-2\delta x^2) = 1 + a^2 + 2a - 4a\delta x^2 = 1 + 1 + 2 - 4\delta x^2 = 4(1-\delta x^2)$$

Plan B: I want to show this

$$\lim_{a\to 0} \frac{(1-a^2)\,[\sin^2(x)]}{1+a^2-2a\cos(2x)} = k\,\delta(x-\pi/2) \qquad x = \omega T_1/2 = \pi\,\omega/\omega_1$$

Change variables to $y = x-\pi/2$

$$\sin(x) = \sin(y+\pi/2) = \cos(y)$$

$$\cos(2x) = \cos(2y + \pi) = -\cos(2y)$$

So I want to show this

$$\lim_{a\to 0} \frac{(1-a^2)\,[\cos^2(y)]}{1+a^2+2a\cos(2y)} = k\,\delta(y)$$

The numerator I feel plays no role, so we really want to show that

$$\lim_{a\to 0} \frac{(1-a^2)}{1+a^2+2a\cos(2y)} = k\,\delta(y)$$

NOW integrate both sides from -ε to ε

$$\lim_{a\to 0} \int_{-\varepsilon}^{\varepsilon} dy \; \frac{(1-a^2)}{1+a^2+2a\cos(2y)} = k$$

$$\lim_{a\to 0} \{ (1-a^2) \int_{-\varepsilon}^{\varepsilon} dy \; [\, 1 + a^2+2a\cos(2y)]^{-1} \; \}$$

$$= \lim_{a\to 0} \{ (1-a^2)\; 2\; (a^2-1)^{-1} \tan^{-1}(\tan\varepsilon\, (a-1)/(a+1)) \}$$

$$= \lim_{a\to 0} \{ \; -\; 2 \; \tan^{-1}(\tan\varepsilon\, (-2)/(a+1)) \}$$

Now as a→-1, the arg of $\tan^{-1}$ approaches -∞, so $\tan^{-1} = -\pi/2$ and we seem then to get

$$= \lim_{a\to 0} \{ \; -2 \; -\pi/2 \; \} = \pi$$

So I have shown then that

$$\lim_{a\to 0} \frac{(1-a^2)}{1+a^2+2a\cos(2y)} = \pi\delta(y)$$

But since periodic, this is really

$$\lim_{a\to 0} \frac{(1-a^2)}{1+a^2+2a\cos(2y)} = \pi\delta(y)$$

But LHS has period π, so we have

$$\lim_{a\to 0} \frac{(1-a^2)}{1+a^2+2a\cos(2y)} = \pi\Sigma_{m=-\infty}^{\infty}\delta(y-m\pi)$$

This is the main idea. Now we can to back to x

$$\lim_{a\to 0} \frac{(1-a^2)\,[\sin^2(x)]}{1+a^2-2a\cos(2x)} = \pi\Sigma_{m=-\infty}^{\infty}\,\delta(x+\pi/2-m\pi)$$

Here is my AMI result with $x = \omega T_1/2$.

$$\frac{\langle|X(\omega)|^2\rangle}{2\pi\delta(0)} = |X_{\mathbf{pulse}}(\omega)|^2 \frac{(1-a^2)\,[\sin^2(\omega T_1/2)]}{1+a^2-2a\cos(\omega T_1)} \qquad a \equiv (1-2p) \qquad \text{AMI}$$

So I have

$$\frac{\langle|X(\omega)|^2\rangle}{2\pi\delta(0)} = |X_{\mathbf{pulse}}(\omega)|^2\, \pi\, \Sigma_{m=-\infty}^{\infty}\, \delta(\omega T_1/2+\pi/2-m\pi)$$

$$= |X_{\mathbf{pulse}}(\omega)|^2\, \pi\, \Sigma_{m=-\infty}^{\infty}(2/T_1)\, \delta(\omega+\pi/T_1-2m\pi/T_1)$$

$$= |X_{\mathbf{pulse}}(\omega)|^2\, \pi\, \Sigma_{m=-\infty}^{\infty}(2/T_1)\, \delta(\omega-(2m-1)\omega_1/2)$$

Let k = 2m-1 so sum ranges

m = -2,-1,0,1,2
k = -5,-3,-1,1,3,5

$$= |X_{\mathbf{pulse}}(\omega)|^2\, \pi\, \Sigma_{\mathbf{k\ odd}}\, (2/T_1)\, \delta(\omega-k\omega_1/2)$$

$$= (2/T_1)\, |X_{\mathbf{pulse}}(\omega)|^2\, 2\pi\, \Sigma_{\mathbf{k\ odd+}}\, \delta(\omega-k\omega_1/2)$$

$$= (2/T_1)\, |X_{\mathbf{pulse}}(\omega)|^2 \sum_{m=1\,3\,5}^{\infty} 2\pi\, \delta(\omega-m\omega_1/2)$$

and this is looking a lot like what I want. For my AMI case I want

$$X_{\mathbf{pulse}}(\omega) = (VT_1)\, \mathrm{sinc}(\omega T_1/2)$$

$$|X_{\mathbf{pulse}}(\omega)|^2 = (VT_1)^2\, \mathrm{sinc}^2(\omega T_1/2)$$

So continue the AMI limit to get

$$= (2/T_1)\, (VT_1)^2\, \mathrm{sinc}^2(\omega T_1/2) \sum_{m=1\,3\,5}^{\infty} 2\pi\, \delta(\omega-m\omega_1/2)$$

$$= (2/T_1)\, (VT_1)^2\, \mathrm{sinc}^2(\pi\, \omega/\omega_1) \sum_{m=1\,3\,5}^{\infty} 2\pi\, \delta(\omega-m\omega_1/2)$$

$$= (2/T_1)\, (VT_1)^2 \sum_{m=1\,3\,5}^{\infty} \mathrm{sinc}^2(\pi\, \omega/\omega_1)\, 2\pi\, \delta(\omega-m\omega_1/2)$$

$$= (2/T_1)\,(VT_1)^2 \sum_{m=1\,3\,5}^{\infty} \mathrm{sinc}^2(\pi\, m/2)\, 2\pi\, \delta(\omega - m\omega_1/2)$$

$$= 2\, V^2\, T_1 \sum_{m=1\,3\,5}^{\infty} \mathrm{sinc}^2(\pi\, m/2)\, 2\pi\, \delta(\omega - m\omega_1/2)$$

This agrees but off by factor of 2.