Comments on Bennet and Davey PhL 5.11.13

1. Comments on Bennet and Davey 1
2. My original source notes from long ago 2
3. Smith "Digital Transmission Systems" 3
4. Library bike trip 13
5. Xiong book stuff 16

1. Comments on Bennet and Davey

The Bennet and Davey theory is based on two different pulse shapes g_1 and g_2, whereas my theory is based on one pulse shape $x_{pulse}(t)$, so very hard to compare results!

I thought this book was the basis of my spectral theory doc, but now that I have the book, it seems to have much less than I thought. Chapter 19 concerns the "statistics of digital signals", and I will try to do some translation between B&D and my stuff.

1. They always use df instead of dω and f in place of ω.

2. My signal x(t) for them is called s(t,T) where $-T < t < T$ so 2T is the length of their finite pulse train.

3. My signal X(ω) form them is S(f,T). Since they use df, their Fourier Transform on page 316 is symmetric.

4. They use $w_s(f,T) \equiv |\, S(f,T)\,|^2 / T$. Roughly this is my $\mathcal{P}(\omega) = |X(\omega)|^2 / 2\pi T$ except my T is half of their T. So w_s is the spectral power density of signal s(t).

4. On page 317 they show my kind of random signal with A and B amplitudes selected at random. Their pulse width is T_s. They in fact use **two different pulse shapes g_1 and g_2** so these would correspond to my two amplitudes of the same x_{pulse} . So roughly $g_1(t) = Ax_{pulse}(t)$ and $g_2(t) = Bx_{pulse}(t)$. But this would only be a special case of the B&D analysis.

5. Their pulse train is written on p 317 as $s(t,T) = \Sigma_{n=-N}^{N} s_n(t)$ where s_n is $g_i(t-nT_s)$. They then assume that g_1 has p and g_2 has 1-p in 19.8.

6. They next define in (19-9),

$$v(t,T) = \Sigma_n [\, p\, g_1(t-nT_s) + (1-p)\, g_2(t-nT_s)\,] = \Sigma_n [\, pA + (1-p)B]x_{pulse}(t-nT_1) \text{ for me.}$$

I am very unclear what this means. For me this is a simple pulse train with y = pA + (1-p)B which seems not very interesting. It is I guess the average of all pulse trains, and they are going to somehow subtract out this average from something. So OK, they define u = s - v, so u is the pulse train with this average pulse train subtracted out. I am very confused.

7. They define a_n in 19-12 in a very strange manner, it is not an amplitude. This is all pretty useless to me! How can I connect it to anything?

$u = s - v$

$u_n(t) = a_n [g_1(t-nT_s) - g_2(t-nT_s)] = a_n [A - B] x_{pulse}(t-nT_1)$

Ouch, I had no idea this would be so bad! Eventually he gets to this: (translate to me)

$w_s(f) = 2f_s p(1-p)(A-B)^2 |X_{pulse}(f)|^2 + f_s^2 [p A + (1-p) B]^2 X_{pulse}(0)^2 \delta(f)$

$+ 2f_s^2 \Sigma_{m=1}^{\infty} | pA + (1-p)B |^2 |X_{pulse}(mf_s)|^2 \delta(f-mf_s)$ STOP!!!

2. My original source notes from long ago

STOP. It just occurs to me that most of my "stuff" came from some other book. Maybe **the AMI thing only came from B&D!** Ouch. Let's go hunt around in my old notes. I find this in contents_refs.doc in my scrambler section (quoted paragraphs from my 1990 notes)

5. D.R.Smith, *Digital Transmission Systems*, VNR, 1985. [Alva Young]

"Chapter 5 of this book is on baseband transmission. It discusses NRZ, NRZI, AMI, CMI, RZ, and other line codes. It mentions the connection between spectral energy and the autocorrelation function, and then quotes without proof the spectral energy density of several line codes. This served as motivation for my Spectral Chapter 6. It has a small amount on (digital) equalization, and Section 5.10 is entitled "data scrambling techniques". This section quotes a few of the results proved in the Scrambler notes. "

6. Bennett and Davey, *Data Transmission*, MH, (1965). [BTS Library]

"This frequently-referenced book is very well written and must be a classic of the field. Chapter 19 is on the statistics of digital signals, which provided hints for Spectral Chapter 6. "

So it is the SMITH book that I was using, arggggggg! I will look now for it on line. This is now in 3rd edition at $140 Amazon. A lot of it seems present in Google books.

http://books.google.com/books?id=4IASeihofskC&printsec=frontcover#v=onepage&q&f=false

I find a download for third edition but I don't believe it, site is

http://www.pdfebookes.com/digital-transmission-systems-PDF4-776203/

Yup, you need some kind of password from http://www.linkbax.com which is probably a virus site. Yup, its one of those ad survey sites which never ends. Cannot find a good download, so let's try Google books again.

3. Smith "Digital Transmission Systems"

This Smith book had some results, but nothing complete, Xiong found later is better, see below.

I will try to quite relevant pieces from the Smith book on Google books.

Chapter on baseband transmission begins on page 258. The TOC is blocked, but I can get that from Amazon,

Contents

Preface vii

Acknowledgments xi

1 Introduction to Digital Transmission Systems 1

1.1 Historical Background 1
1.2 Present-Day Digital Transmission 6
1.3 Digital Transmission Standards 11
1.4 Advantages of Digital Transmission 15
1.5 A Simplified Digital Transmission System 16
1.6 Summary 17

2 Principles of System Design 21

2.1 General Plan 21
2.2 Transmission Services 22
2.3 Hypothetical Reference Circuits 33
2.4 Performance Objectives 38

2.5 Summary 77
Problems 81

3 Analog-to-Digital Conversion Techniques 84
3.1 Introduction 84
3.2 Pulse Code Modulation 85
3.3 Differential PCM and Delta Modulation 112
3.4 Adaptive Differential Pulse Code Modulation 123
3.5 Subband Coding 131
3.6 Transform Coding 134
3.7 Comparison of Waveform Coders 136
3.8 Voice Coders (Vocoders) 144
3.9 Hybrid Coders 150
3.10 Voice over IP 155
3.11 Video Coding 156
3.11 Summary 160
Problems 167

4 Time-Division Multiplexing 177
4.1 Introduction 177
4.2 Synchronous Multiplexing 180
4.3 Asynchronous Multiplexing 200
4.4 Plesiochronous Digital Hierarchies 214
4.5 Statistical Multiplexing and Speech Interpolation 236
4.6 Summary 246
Problems 250

5 Baseband Transmission 258
5.1 Introduction 259
5.2 Types of Binary Coding 261
5.3 Power Spectral Density of Binary Codes 269
5.4 Error Performance of Binary Codes 275
5.5 Block Line Codes 280
5.6 Pulse Shaping and Intersymbol Interference 285
5.7 Multilevel Baseband Transmission 289
5.8 Partial Response Coding 291
5.9 Eye Patterns 301
5.10 Equalization 301
5.11 Data Scrambling Techniques 309
5.12 Spread Spectrum 316
5.13 Access Techniques 323
5.14 Error Detection and Correction Coding 327

Copyrighted Material

5.15 Summary 342
Problems 346

6 Digital Modulation Techniques 363
6.1 Introduction 363
6.2 Binary Amplitude-Shift Keying (ASK) 370
6.3 Binary Frequency-Shift Keying (FSK) 374
6.4 Binary Phase-Shift Keying (BPSK) 377
6.5 Comparison of Binary Modulation Systems 380
6.6 *M*-ary FSK 381
6.7 *M*-ary PSK 386
6.8 Quadrature Amplitude Modulation (QAM) 394
6.9 Offset QPSK, $\pi/4$ QPSK, and Minimum-Shift Keying 399
6.10 Quadrature Partial Response (QPR) 408
6.11 Orthogonal Frequency Division Multiplexing (OFDM) 412
6.12 Trellis Coded Modulation 415
6.13 Summary 424
Problems 429

7 Digital Cable Systems 437
7.1 Introduction 437
7.2 Cable Characteristics 438
7.3 Regenerative Repeaters 444
7.4 Clock Recovery and Jitter 445
7.5 Crosstalk 451
7.6 Performance of Digital Subscriber Lines 454
7.7 Error Performance Tandem Repeaters 455
7.8 Repeater Spacing in Multipair Cable Systems 456
7.9 Repeater Spacing in Coaxial Cable Systems 464
7.10 Implementation Considerations 466
7.11 Summary 468
Problems 470

8 Fiber Optic Transmission Systems 472
8.1 Introduction 472
8.2 Fiber Optic Cable 474
8.3 Optical Sources 484
8.4 Optical Detectors 489
8.5 Optical Amplifiers 496
8.6 Wavelength-Division Multiplexing 501

8.7 Fiber Optic Link Design 508
8.8 SONET 520
8.9 Synchronous Digital Hierarchy 534
8.10 Summary 537
Problems 541

9 Digital Radio Systems **547**
9.1 Introduction 547
9.2 Propagation 548
9.3 Fading 566
9.4 Diversity 579
9.5 Antennas 585
9.6 Frequency Allocations and Interference Effects 590
9.7 Radio Link Calculations 592
9.8 Summary 597
Problems 600

10 Network Timing and Synchronization **612**
10.1 Introduction 612
10.2 Time Standards 614
10.3 Frequency Sources and Clocks 616
10.4 Network Synchronization Techniques 629
10.5 Time and Frequency Dissemination Systems 642
10.6 Examples of Network Synchronization Schemes 652
10.7 Summary 656
Problems 660

11 Transmission System Testing, Monitoring, and Control **662**
11.1 Introduction 662
11.2 Testing Techniques 663
11.3 Performance Monitoring Techniques 674
11.4 Fault Isolation 681
11.5 Monitoring and Control Systems 685
11.6 Digital Cross-Connect Systems 693
11.7 Summary 669
Problems 702

12 Digital Transmission Over Analog Networks **705**
12.1 Introduction 705
12.2 Transmission Parameters 706
12.3 Conditioning 711
12.4 Voice-Band Modems 713

12.5 V.90 Modems 720
12.6 Digital Subscriber Lines 722
12.7 Cable Modems 732
12.7 Summary 739
Problems 742

13 Digital Transmission Networks 745
13.1 Introduction 745
13.2 Private Networks 746
13.3 Personal Communication Networks 751
13.4 Wide Area Networks 761
13.5 Optical Networks 776
13.6 Summary 793
Problems 795

Index 799

For my purposes, only Chapter 5 on Baseband is relevant. This is pp 258-346. So how much of these 80 pages can be seen in Google books? It is perforated with missing pages, but here are a few things I can extract, [a few snippets available from **Google Books** version of the **Smith** book:]

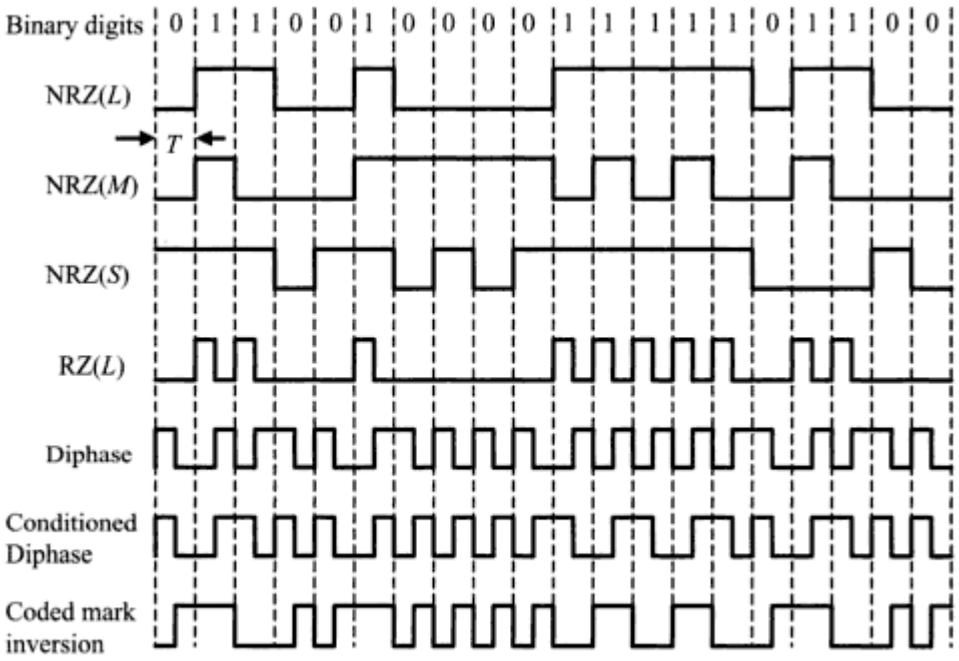


FIGURE 5.1 Binary Coding Waveforms

5.3 Power Spectral Density of Binary Codes

The power spectral density of a baseband code describes two important transmission characteristics: required bandwidth and spectrum shaping. The bandwidth available in a transmission channel is described by its frequency response, which typically indicates limits at the high or low end. Further bandwidth restriction may be imposed by a need to stack additional signals into a given channel. Spectrum shaping can help minimize interference from other signals or noise. Conversely, shaping of the signal spectrum can allow other signals to be added above or below the signal bandwidth.

In the following derivations we assume that the digital source produces 0's and 1's with equal probability. Each bit interval is $(0, T)$ and each corresponding waveform is limited to an interval of time equal to T.

Power Spectral Density

Derivation of the power spectral density is facilitated by starting with the autocorrelation function, which is defined for a random process $x(t)$ as

$$R_x(\tau) = \mathrm{E}[x(t)x(t+\tau)] \tag{5.3}$$

where E[·] represents the expected value or mean. The power spectral density describes the distribution of power versus frequency, and is given by the Fourier transform of the autocorrelation function

$$S_x(f) = \mathscr{F}[R_x(\tau)] = \int_{-\infty}^{\infty} R_x(\tau)e^{-j2\pi f\tau}d\tau \tag{5.4}$$

In some cases $S_x(f)$ may be more conveniently derived directly from the Fourier transform of the signal. Assuming the signal $x(t)$ to be zero outside the range $-T/2$ to $T/2$, then the corresponding frequency domain signal is given by

$$X(f) = \int_{-\infty}^{\infty} x(t)\, e^{-j2\pi ft} dt$$

$$\int_{-T/2}^{T/2} x(t)\, e^{-j2\pi ft} dt \tag{5.5}$$

By Parseval's theorem the average power across a 1-ohm load over the time interval $-T/2$ to $T/2$ is given by

$$P = \frac{1}{T} \int_{-T/2}^{T/2} x^2(t) dt$$

$$= \frac{1}{T} \int_{-\infty}^{\infty} |X(f)|^2 df \tag{5.6}$$

But from the definition of power spectral density, the average power can also be expressed as

$$P = \int_{-\infty}^{\infty} S_x(f) df \tag{5.7}$$

where $S_x(f)$ is in units of watts per hertz and P is in units of watts. From (5.6) and (5.7) we obtain

$$S_x(f) = \frac{|X(f)|^2}{T} \tag{5.8}$$

5.3.1 Nonreturn-to-Zero (NRZ)

With polar NRZ transmission, we assume that the two possible signal levels are V and $-V$ and that the presence of V or $-V$ in any 1-bit interval is statistically independent of that in any other bit interval. The autocorrelation function for NRZ is well known and is given by

$$R_Q(\tau) = \begin{cases} V^2(1 - |\tau|/T) & |\tau| < T \\ 0 & |\tau| > T \end{cases} \tag{5.9}$$

Using the Fourier transform, we find the corresponding power spectral density to be

$$S_Q(f) = V^2 T \left(\frac{\sin \pi fT}{\pi fT} \right)^2 \tag{5.10}$$

// agrees with Xiong

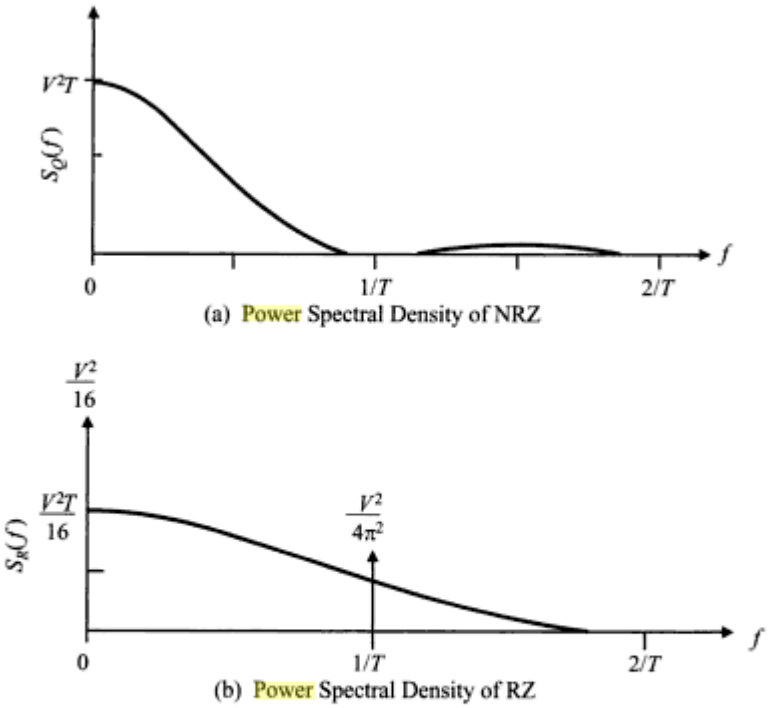


FIGURE 5.7 Power Spectral Density of NRZ and RZ Coding

This is shown in Figure 5.7a. With most of its energy in the lower frequencies, NRZ is often a poor choice for baseband transmission due to the presence of interfering signals around dc. Moreover, clock recovery at the receiver is complicated by the fact that random NRZ data do not exhibit discrete spectral components.

5.3.2 Return-to-Zero (RZ)

In the RZ coding format, signal values are

$$f_1(t) = \begin{cases} V & 0 \le t \le T/2 \\ 0 & T/2 < t < T \end{cases}$$

$$f_2(t) = 0 \qquad (5.11)$$

The power spectral density can be found by first dividing the RZ signal into two components. The first component is a periodic pulse train of amplitude

but p 272,273 are missing, continuing:

5.3.4 *Bipolar*

The bipolar coding format uses three amplitudes: V, 0, $-V$. The bipolar waveforms are given by

$$f_1(t) = 0$$

$$f_2(t) = \begin{cases} \left.\begin{matrix} V & 0 < t \le \alpha T \\ 0 & \alpha T < t \le T \end{matrix}\right\} & \textit{first 1} \\ \\ \left.\begin{matrix} -V & 0 < t \le \alpha T \\ 0 & \alpha T < t \le T \end{matrix}\right\} & \textit{next 1} \end{cases} \tag{5.20}$$

where $\alpha = 1$ represents nonreturn-to-zero (100 percent duty cycle) and $\alpha = 1/2$ represents return-to-zero (50 percent duty cycle). The power spectral density of bipolar signaling is identical to that of *twinned binary* [1], which can be generated from NRZ by delaying it a T, subtracting it from the original, and dividing by 2 to provide proper amplitude scaling-that is,

$$B(t) = \frac{Q(t) - Q(t - \alpha T)}{2} \tag{5.21}$$

Taking the Fourier transform of both sides of (5.21) we have

$$B(f) = \frac{Q(f)}{2}(1 - e^{-j2\pi f \alpha T}) \tag{5.22}$$

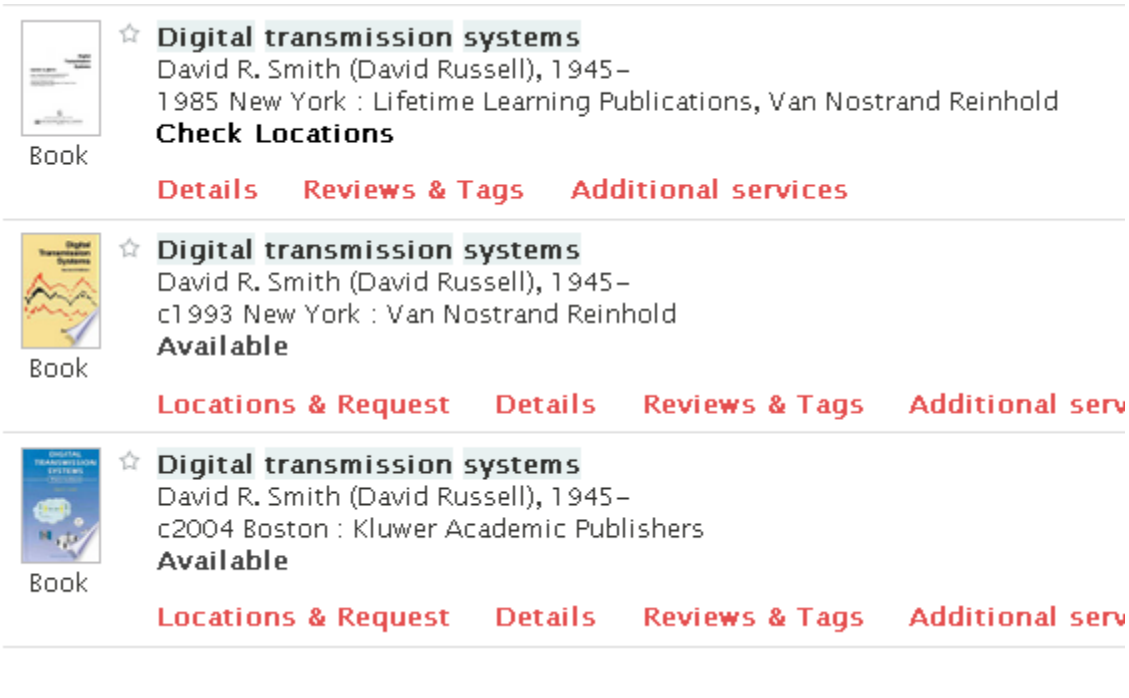
From the definition of power spectral density (5.8),

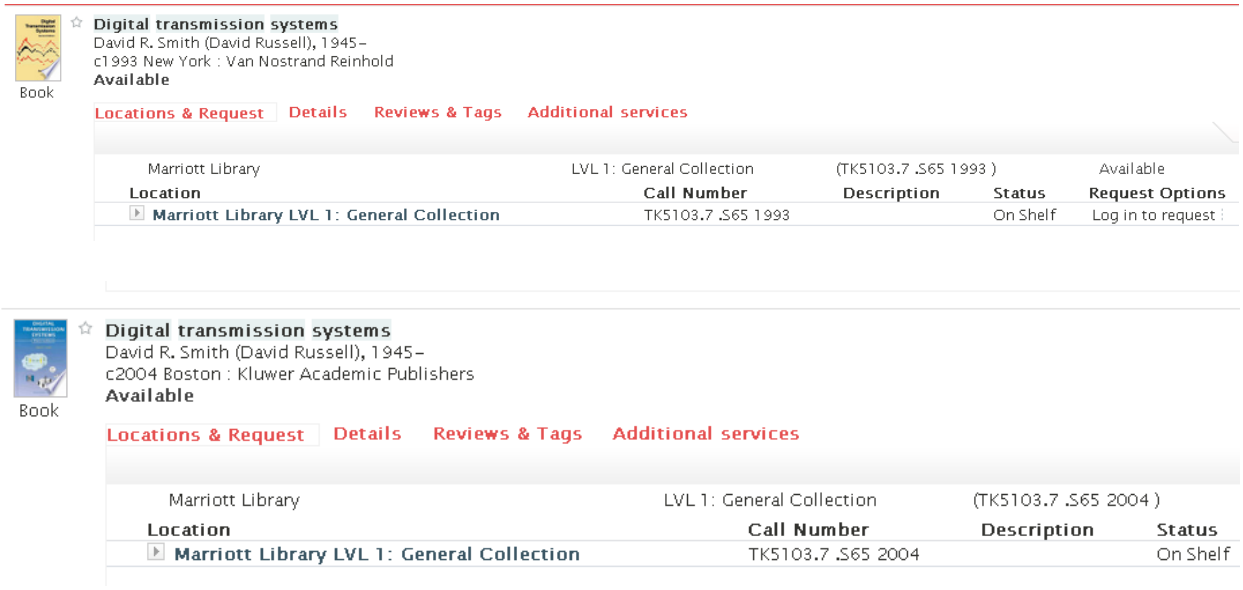
$$\begin{aligned} S_B(f) &= \frac{|Q(f)|^2}{4T} \left|1 - e^{-j2\pi f \alpha T}\right|^2 \\ &= S_Q(f) \sin^2 \pi f \alpha \mathrm{T} \end{aligned} \tag{5.23}$$

The resulting power spectral density is shown in Figure 5.9 for both bipolar return-to-zero and bipolar nonreturn-to-zero. The class of bipolar codes that use filling sequences to substitute for strings of zeros have power spectral densities that are similar to simple bipolar, but each member of this class differs slightly as shown in Figure 5.9. Note the lack of discrete lines in any of the bipolar power spectral densities. The bandwidth occupancy of bipolar NRZ and bipolar RZ is similar to NRZ and RZ, respectively, although bipolar has relatively low power at low frequencies and zero power at dc—another advantage of bipolar over NRZ coding.

That's about all there is!

Marriott Smith: the three editions are 1985, 1993 and 2004.





SO they claim that the two newest editions are "on the shelf" !

But where is Smith getting his results FROM? Answer not clear, the chapter does have a page of pretty narrow topic references. So I am not immediately led to another book.

4. Library bike trip to look up Smith, liked Xiong better.

SMITH STUFF

After taking these photos, I was able to download the Xiong book (minus a few pages).

Library Bike Trip. I took my camera to Marriott and found lots of good books. The 2004 Smith book was there, and that is the first set of photos. Here are my results from that book: [Smith]

NRZ:

$$R_Q(\tau) = \begin{cases} V^2(1 - |\tau|/T) & |\tau| < T \\ 0 & |\tau| > T \end{cases}$$

$$S_Q(f) = V^2 T \left(\frac{\sin \pi f T}{\pi f T} \right)^2$$

RZ:

$$f_1(t) = \begin{cases} V & 0 \le t \le T/2 \\ 0 & T/2 < t < T \end{cases}$$

$$f_2(t) = 0$$

The random component may be handled in a manner similar to the NRZ case, yielding a continuous power spectral density:

$$S_r(f) = \frac{V^2 T}{16} \left[\frac{\sin(\pi f T/2)}{\pi f T/2} \right]^2 \quad (5.15)$$

The periodic component may be evaluated by standard Fourier series analysis as

$$S_p(f) = \sum_{n=-\infty}^{\infty} \frac{V^2}{16} \left[\frac{\sin(n\pi/2)}{n\pi/2} \right]^2 \delta\left(f - \frac{n}{T}\right) \quad (5.16)$$

The power spectral density for RZ is simply the sum of (5.15) and (5.16). This is shown in Figure 5.7b. Note that the first zero crossing of RZ is at twice the frequency as that for NRZ, indicating that RZ requires twice the bandwidth of NRZ. However, the discrete spectral lines of RZ permit simpler bit synchronization.

// sum agrees with Xiong

Bipolar: (with α = 1 this is my AMI I think) (yes sometimes this IS called Bipolar)

5.3.4 *Bipolar*

The bipolar coding format uses three amplitudes: V, 0, $-V$. The bipolar waveforms are given by

$$f_1(t) = 0$$

$$f_2(t) = \begin{cases} V & 0 < t \le \alpha T \\ 0 & \alpha T < t \le T \end{cases} \text{ first 1}$$

$$\begin{cases} -V & 0 < t \le \alpha T \\ 0 & \alpha T < t \le T \end{cases} \text{ next 1} \qquad (5.20)$$

$$S_B(f) = \frac{|Q(f)|^2}{4T} |1 - e^{-j2\pi f \alpha T}|^2$$

$$= S_Q(f) \sin^2 \pi f \alpha T$$

Diphase: (in the graph below he suggests this is Manchester) .

5.3.3 *Diphase*

In the diphase format, the signal values are given by (5.2). The power spectral density may be found from (5.8) by first determining the Fourier transform

but I don't have (5.2). The spectrum is this:

As determined by W. R. Bennett [6], the power spectral density depends on the difference between the Fourier transforms of the two pulses:

$$S_D(f) = \frac{1}{T} |F_1(f) - F_2(f)|^2 \qquad (5.18)$$

or from (5.17)

$$S_D(f) = V^2 T \left[\frac{\sin^4(\pi f T/2)}{(\pi f T/2)^2} \right] \qquad (5.19)$$

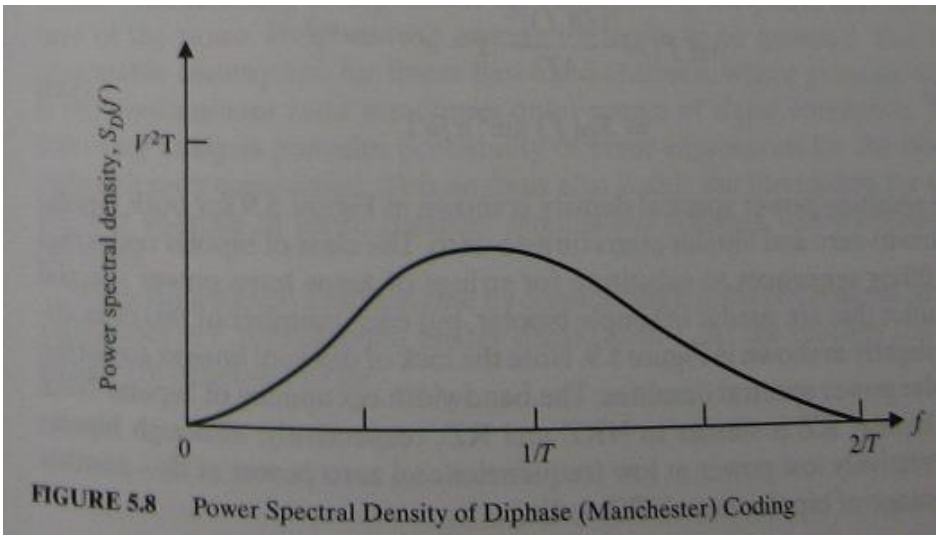


FIGURE 5.8 Power Spectral Density of Diphase (Manchester) Coding

I then switched to another book which I think was Xiong, Digital Modulation Techniques.

5. Xiong book stuff

Publisher: Artech House Publishers; 1st edition (January 31, 2000)

Language: English

ISBN-10: 0890069700

ISBN-13: 978-0890069707

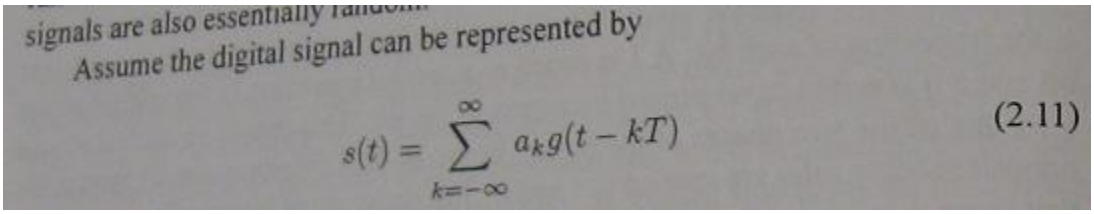
Digital Modulation Techniques (Artech House Telecommunications Library) [Hardcover]

Fuqin Xiong (Author)

★★★★★ (1 customer review)

I was able to **download** this from 4shared and confirm the page numbers of my photos. Page 28 of the download is blank, but I have a photo.

Here are results from that source (these are on page 28)



signals are also essentially

Assume the digital signal can be represented by

$$s(t) = \sum_{k=-\infty}^{\infty} a_k g(t - kT) \qquad (2.11)$$

In Appendix A (A.16) we show that the power spectral density

$$\Psi_s(f) = \frac{|G(f)|^2}{T} \sum_{n=-\infty}^{\infty} R(n) e^{-jn\omega T} \tag{2.12}$$

where $\omega = 2\pi f$. $G(f)$ is the Fourier transform of $g(t)$ and $R(n)$ is the autocorrelation function of random sequence $\{a_k\}$, defined as $R(n) = E\{a_k a_{k+n}\}$, where $E\{x\}$ is the probabilistic average of x. Equation (2.12) shows that the PSD of a digitally modulated signal is not only determined by its symbol function but also is affected by the autocorrelation function of the data sequence.

The rest I take from the download,

where σ_a^2 is the variance and m_a is the mean of the sequence $\{a_k\}$. Using the Poisson sum formula, the PSD expression can be written as (see Appendix A, (A.17))

$$\Psi_s(f) = \frac{|G(f)|^2}{T} \left[\sigma_a^2 + m_a^2 R_b \sum_{n=-\infty}^{\infty} \delta(f - nR_b) \right] \tag{2.14}$$

where $R_b = 1/T$ is the data bit rate.

where σ_a^2 is the variance and m_a is the mean of the sequence $\{a_k\}$. Using the Poisson sum formula, the PSD expression can be written as (see Appendix A, (A.17))

$$\Psi_s(f) = \frac{|G(f)|^2}{T} \left[\sigma_a^2 + m_a^2 R_b \sum_{n=-\infty}^{\infty} \delta(f - nR_b) \right] \tag{2.14}$$

where $R_b = 1/T$ is the data bit rate.

NRZ formats' symbol function is a square pulse with amplitude A in the interval $[0, T]$. which can be expressed as

$$g(t) = \begin{cases} A, & 0 \le t \le T \\ 0, & \text{elsewhere} \end{cases} \tag{2.16}$$

Its Fourier transform can be easily found as

$$\begin{aligned} G(f) &= AT\left(\frac{\sin \pi fT}{\pi fT}\right) e^{-j\pi fT} \\ &= AT\text{sinc}(\pi fT)e^{-j\pi fT} \end{aligned} \tag{2.17}$$

where $sinc(x) = sin(x)/x$ is the *sinc function*. Next we need to find the autocorrelation function $R(n)$ of the binary data sequence $\{a_k\}$. For this waveform

etc etc and then

$$\Psi_s(f) = A^2T\left(\frac{\sin \pi fT}{\pi fT}\right)^2. \quad \text{(NRZ)} \tag{2.20}$$

We mentioned that all three NRZ waveforms can be made unipolar. Assume the pulse amplitude is A, then there is a dc component of A/2 in the signals and it appears

$$\Psi_s(f) = \frac{A^2T}{4}\left(\frac{\sin \pi fT}{\pi fT}\right)^2 + \frac{A^2}{4}\delta(f). \quad \text{(unipolar NRZ)} \tag{2.25}$$

For RZ formats, the pulse function is a square pulse with half-bit duration

$$g(t) = \begin{cases} A, & 0 \le t \le \frac{T}{2} \\ 0, & \text{elsewhere} \end{cases} \tag{2.26}$$

The corresponding Fourier transform is

$$G(f) = \frac{AT}{2}\left(\frac{\sin \pi fT/2}{\pi fT/2}\right) e^{-j\pi fT/2} \tag{2.27}$$

For *polar RZ* waveform

$$a_k = \begin{cases} 1, & \text{for binary 1}, \quad p_1 = 1/2 \\ -1, & \text{for binary 0}, \quad p_0 = 1/2 \end{cases} \tag{2.28}$$

which is the same as polar NRZ. Thus

$$R(n) = \begin{cases} 1, & n = 0 \\ 0, & n \neq 0 \end{cases} \tag{2.29}$$

Substituting (2.27) and (2.29) into (2.12), the result is

$$\Psi_s(f) = \frac{A^2T}{4}\left(\frac{\sin \pi fT/2}{\pi fT/2}\right)^2, \quad \text{(Polar RZ)} \tag{2.30}$$

For *unipolar RZ* format the symbol pulse is the same as in (2.26). The data sequence, its mean and variance are the same as those of unipolar NRZ as given in (2.22), (2.23) and (2.24). Substituting $G(f)$ in (2.27), m_a and σ_a^2 into (2.14) we have

$$\Psi_s(f) = \frac{A^2T}{16}\left(\frac{\sin \pi fT/2}{\pi fT/2}\right)^2 \left[1 + R_b \sum_{k=-\infty}^{\infty} \delta(f - nR_b)\right], \text{ (unipolar RZ)} \tag{2.31}$$

2.3.4 PSD of Biphase Codes

For *Bi-Φ-L (Manchester),* the symbol function is half-positive and half-negative pulse defined by

$$g(t) = \begin{cases} A, & 0 \le t \le \frac{T}{2} \\ -A, & \frac{T}{2} \le t \le T \\ 0, & \text{elsewhere} \end{cases} \tag{2.36}$$

The Fourier transform of $g(t)$ is

$$G(f) = AT \left(\frac{\sin \pi fT/2}{\pi fT/2} \right) (\sin \pi fT/2)\, je^{-j\omega T/2} \tag{2.37}$$

The data probability distribution is

$$a_k = \begin{cases} 1, & \text{for binary 1, } p_1 = 1/2 \\ -1, & \text{for binary 0, } p_0 = 1/2 \end{cases}$$

which is the same as that of NRZ. We have shown that $R(n) = 1$ for $n = 0$ and $R(n) = 0$ for $n \neq 0$ (2.19). Using (2.12) we obtain

$$\Psi_s(f) = A^2T \left(\frac{\sin \pi fT/2}{\pi fT/2} \right)^2 \sin^2 \pi fT/2. \quad \text{(Bi-Φ-L)} \tag{2.38}$$

This PSD is shown in Figure 2.4(c) where we set $A = 1$ for unity symbol energy.

I then copied some pages from the Xiong appendix. There he comes up with this general formula, '

$$
\begin{aligned}
\Psi_s(f) &= \lim_{N\to\infty} \frac{1}{(2N+1)T} E\{|A_N(f)|^2\} \\
&= \lim_{N\to\infty} \frac{|G(f)|^2}{(2N+1)T} E\left\{\left|\sum_{n=-N}^{N} a_n e^{-j\omega nT}\right|^2\right\} \\
&= \lim_{N\to\infty} \frac{|G(f)|^2}{(2N+1)T} E\left\{\left(\sum_{n=-N}^{N} a_n e^{-j\omega nT}\right)\left(\sum_{n=-N}^{N} a_n e^{-j\omega nT}\right)^*\right\} \\
&= |G(f)|^2 \lim_{N\to\infty} \frac{1}{(2N+1)T} \sum_{n=-N}^{N}\sum_{m=-N}^{N} E\{a_n a_m\} e^{j(m-n)\omega T}
\end{aligned}
$$

$$
= |G(f)|^2 \lim_{N\to\infty} \frac{1}{(2N+1)T} \sum_{n=-N}^{N}\sum_{k=n+N}^{n-N} R(k) e^{-jk\omega T}
$$

$$= |G(f)|^2 \lim_{N\to\infty} \frac{1}{(2N+1)T} \sum_{n=-N}^{N} \sum_{k=n+N}^{n-N} R(k)e^{-jk\omega T}$$

where $k = n-m$, $E\{a_n a_m\} = E\{a_n a_{n-k}\} = R(k)$ is the autocorrelation function of the data bits. Note that $R(-k) = R(k)$. Realizing the inner summations become the same regardless of the value of index n when $N \to \infty$, we obtain

$$\begin{aligned}\Psi_s(f) &= \frac{|G(f)|^2}{T} \lim_{N\to\infty} \left(\frac{2N+1}{(2N+1)} \sum_{k=n+N}^{n-N} R(k)e^{-jk\omega T} \right) \\ &= \frac{|G(f)|^2}{T} \sum_{k=\infty}^{-\infty} R(k)e^{-jk\omega T}\end{aligned}$$

Equivalently this is

$$\Psi_s(f) = \frac{|G(f)|^2}{T} \sum_{k=-\infty}^{\infty} R(k)e^{-jk\omega T} \qquad \text{(A.16)}$$

Now we discuss two possible cases of $R(k)$.

A.3.1 Case 1: Data Symbols Are Uncorrelated

Assume a_n has a mean of $E\{a_n\} = m_a$ and a variance of σ_a^2, then

$$\begin{aligned}R(k) &= \begin{cases} E\{a_n^2\}, & k = 0 \\ E\{a_n\}E\{a_{n-k}\}, & k \neq 0 \end{cases} \\ &= \begin{cases} \sigma_a^2 + m_a^2, & k = 0 \\ m_a^2, & k \neq 0 \end{cases}\end{aligned}$$

Substitute this for $R(k)$ in (A.16) we have

$$\Psi_s(f) = \frac{|G(f)|^2}{T} \left(\sigma_a^2 + m_a^2 \sum_{k=-\infty}^{\infty} e^{-jk\omega T} \right)$$

By revoking the Poisson sum formula ([1] p.62)

$$\sum_{k=-\infty}^{\infty} e^{-jk\omega T} = \frac{1}{T}\sum_{k=-\infty}^{\infty} \delta(f - \frac{k}{T})$$

I don't think I gave this formula a name did I?

where $\delta(f)$ is the impulse function, we have

$$\Psi_s(f) = \frac{|G(f)|^2}{T}\left(\sigma_a^2 + \frac{m_a^2}{T}\sum_{k=-\infty}^{\infty}\delta(f - \frac{k}{T})\right) \qquad \text{(A.17)}$$

That is, for uncorrelated data

$$\Psi_s(f) = \underbrace{\frac{\sigma_a^2|G(f)|^2}{T}}_{\text{continuous spectrum}} + \underbrace{(\frac{m_a}{T})^2\sum_{k=-\infty}^{\infty}|G(\frac{k}{T})|^2\delta(f - \frac{k}{T})}_{\text{discrete spectrum}} \qquad \text{(A.18)}$$

A.3.2 Case 2: Data Symbols Are Correlated

For the general case where there is correlation between that data, let $\widetilde{a}_n$ be the corresponding data that have been normalized to have unity variance and zero mean, that is $\widetilde{a}_n = (a_n - m_a)/\sigma_a$, thus $a_n = \sigma_a \widetilde{a}_n + m_a$, then

$$\begin{aligned} R(k) &= E\{a_n a_{n-k}\} = E\{(\sigma_a \widetilde{a}_n + m_a)(\sigma_a \widetilde{a}_{n-k} + m_a)\} \\ &= \sigma_a^2 E\{\widetilde{a}_n \widetilde{a}_{n-k}\} + \underbrace{E\{\sigma_a m_a \widetilde{a}_n\}}_{0} + \underbrace{E\{\sigma_a m_a \widetilde{a}_{n-k}\}}_{0} + E\{m_a^2\} \\ &= \sigma_a^2 \rho(k) + m_a^2 \end{aligned}$$

where

$$\rho(k) = E\{\widetilde{a}_n \widetilde{a}_{n-k}\}$$

is the autocorrelation coefficient. Thus for correlated data

$$\begin{aligned} \Psi_s(f) &= \frac{|G(f)|^2}{T}\left(\sigma_a^2 \sum_{k=-\infty}^{\infty} \rho(k) e^{-jk\omega T} + m_a^2 \sum_{k=-\infty}^{\infty} e^{-jk\omega T}\right) \\ &= \underbrace{\sigma_a^2 R_d |G(f)|^2 w_p(f)}_{\text{continuous spectrum}} + \underbrace{(m_a R_d)^2 \sum_{k=-\infty}^{\infty} |G(kR_d)|^2 \delta(f - kR_d)}_{\text{discrete spectrum}} \end{aligned} \tag{A.20}$$

Notice that all authors use "power spectral density" PSD and not "spectral power density".

And THAT is the end of my slide show. I looked in a very new Proakis edition and an older one but could find nothing really on this subject. All in all, this was a good bike library trip.