Correlation stuff PhL 3.22.13

Wolfram: note there are two different symbols, star and asterisk.

http://mathworld.wolfram.com/Cross-Correlation.html

Cross-Correlation

The cross-correlation of two complex functions $f(t)$ and $g(t)$ of a real variable t, denoted $f \star g$ is defined by

$$f \star g \equiv \overline{f}(-t) * g(t), \tag{1}$$

where $*$ denotes convolution and $\overline{f}(t)$ is the complex conjugate of $f(t)$. Since convolution is defined by

$$f * g = \int_{-\infty}^{\infty} f(\tau)\, g(t-\tau)\, d\tau, \tag{2}$$

it follows that

$$[f \star g](t) \equiv \int_{-\infty}^{\infty} \overline{f}(-\tau)\, g(t-\tau)\, d\tau. \tag{3}$$

Letting $\tau' \equiv -\tau$, $d\tau' = -d\tau$, so (3) is equivalent to

$$f \star g = \int_{\infty}^{-\infty} \overline{f}(\tau')\, g(t+\tau')\, (-d\tau') \tag{4}$$

$$= \int_{-\infty}^{\infty} \overline{f}(\tau)\, g(t+\tau)\, d\tau. \tag{5}$$

The cross-correlation satisfies the identity

$$(g \star h) \star (g \star h) = (g \star g) \star (h \star h). \tag{6}$$

If f or g is even, then

$$f \star g = f * g, \tag{7}$$

where $*$ again denotes convolution.

This is my general area of interest, but things are missing and wrong. First, the last claim would require that f be real and even, unless "even" has some special meaning for a complex function.

Wiki defines things like this

http://en.wikipedia.org/wiki/Cross-correlation

For continuous functions, f and g, the cross-correlation is defined as:

$$(f \star g)(t) \stackrel{\text{def}}{=} \int_{-\infty}^{\infty} f^*(\tau)\ g(t+\tau)\, d\tau,$$

and again the star is used. OK, this agrees with Wolfram just take τ→-τ. So this looks the same as convolution but the sign of t is different and f* is appearing. Do I have a star character?

star operator	★	U+22C6

Do I have such a character? ⋆

⋆8902⋆22C6 STAR OPERATOR

My Lucida Unicode font has it. I have to make it larger to have it be right. Then

$$(f \star g)(t) = \int_{-\infty}^{\infty} dt'\, f^*(t')g(t+t')dt'$$

$$(f^* \star g)(-t) = \int_{-\infty}^{\infty} dt'\, f(t')g(t'-t)dt' = (f * g)(t)$$

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Recall the convolution theorem (3.6)

$$a(t) = \int_{-\infty}^{\infty} dt'\, b(t-t')c(t') \quad \Leftrightarrow \quad A(\omega) = B(\omega)\, C(\omega) \qquad (3.6)$$

The equation on the left is often written

$$a(t) = (b * c)(t)$$

and one says that a is the convolution of b with c.

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$$[c(t) \star b(t)](t) = \int_{-\infty}^{\infty} dt'\, c^*(t')b(t+t') = \int_{-\infty}^{\infty} dt''\, c^*(-t'')b(t-t'') = [c^*(-t) * b(t)](t)$$

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<u>The Convolution Theorem:</u>

$$a(t) = \int_{-\infty}^{\infty} dt'\, b(t-t')c(t') \quad \Leftrightarrow \quad A(\omega) = B(\omega)\, C(\omega) \qquad (3.6)$$

<u>Autocorrelation Function:</u>

$$r_b(t) \equiv \int_{-\infty}^{\infty} dt'\, b(t')\, b(t' + t) = \int_{-\infty}^{\infty} dt'\, b(t')\, b(t' - t) \qquad (32.1)$$

Cross Correlation Function of b with itself

$$[b(t) \star b(t)](t) = \int_{-\infty}^{\infty} dt'\, b^*(t') b(t+t')$$

This, if b is a real function,

If