Verification of Line Codes PhL 5.12.13

1. The NRZ waveforms

For my unipolar NRZ I get $x = \omega/\omega_1$

$$\langle \mathcal{P}(\omega) \rangle = (V^2/\omega_1)\, \mathrm{sinc}^2(\pi x)\, [\, p(1-p) + p^2\, \delta(x)\,] \qquad \text{// unipolar NRZ} \qquad (36.3)$$

and for my bipolar NRZ I get

$$\langle \mathcal{P}(\omega) \rangle = (V^2/\omega_1)\, \mathrm{sinc}^2(\pi x)\, [\, 4p(1-p) + (1-2p)^2 \delta(x)\,] \qquad \text{// bipolar NRZ} \qquad (36.6)$$

If I set p = 1/2 in these results I get

$$\langle \mathcal{P}(\omega) \rangle = (V^2/\omega_1)\, \mathrm{sinc}^2(\pi x)\, [1/4 + 1/4\, \delta(x)\,] \qquad \text{// unipolar NRZ} \qquad (36.3)$$

$$\langle \mathcal{P}(\omega) \rangle = (V^2/\omega_1)\, \mathrm{sinc}^2(\pi x) \qquad \text{// bipolar NRZ} \qquad (36.6)$$

Smith defines his NRZ in terms of voltages V and -V, so his is bipolar NRZ. My result in f space is

$$\langle P(f) \rangle / 2\pi = (V^2/\omega_1)\, \mathrm{sinc}^2(\pi x) = V^2 T_1/(2\pi) * \mathrm{sinc}^2(\pi x)$$

$$\langle P(f) \rangle = V^2 T_1\, \mathrm{sinc}^2(\pi x) = V^2 T_1\, \mathrm{sinc}^2(\pi\, f T_1) \qquad x = \omega/\omega_1 = T_1/T = f T_1$$

This is in agreement with Smith which I now quote

$$S_Q(f) = V^2 T \left(\frac{\sin \pi f T}{\pi f T} \right)^2 \qquad (5.10)$$

and this agrees also with Xiong which I quote

$$\Psi_s(f) = A^2 T \left(\frac{\sin \pi f T}{\pi f T} \right)^2 . \qquad \text{(NRZ)} \qquad (2.20)$$

Going back to my unipolar, I get for p = 1/2

$$\langle \mathcal{P}(\omega) \rangle = (V^2/\omega_1)\, \mathrm{sinc}^2(\pi x)\, [1/4 + 1/4\, \delta(x)\,] \qquad \text{// unipolar NRZ} \qquad (36.3)$$

$$\langle P(f) \rangle / 2\pi = V^2 T_1/(2\pi) * \mathrm{sinc}^2(\pi x)\, [\, 1/4 + 1/4\, \delta(x)\,]$$

$$\langle P(f) \rangle = V^2 T_1\, \mathrm{sinc}^2(\pi\, f T_1)\, [\, 1/4 + 1/4\, \delta(f T_1)\,] / (2\pi)$$

$= \quad V^2 T_1 \, \mathrm{sinc}^2(\pi f T_1) \, [\, 1/4 + 1/4 \, (1/T_1)\delta(f) \,]$

$= \quad (1/4) \, V^2 T_1 \, \mathrm{sinc}^2(\pi f T_1) \; + \; 1/4 \, V^2 T_1 (1/T_1)\delta(f)$

$= \quad (1/4) \, V^2 T_1 \, \mathrm{sinc}^2(\pi f T_1) \; + \; (1/4) \, V^2 \delta(f)$

This agrees exactly with Xiong,

$$\Psi_s(f) = \frac{A^2 T}{4} \left(\frac{\sin \pi f T}{\pi f T} \right)^2 + \frac{A^2}{4} \delta(f). \qquad \text{(unipolar NRZ)} \qquad (2.25)$$

so I am happy with my NRZ verifications.

2. The RZ waveforms

My RZ is unipolar and the pulse fills the left half of the gap as in my Fig 36.8. My result is

$$\langle \mathcal{P}(\omega) \rangle \; = \; (V/2)^2 \, (1/\omega_1) \, \mathrm{sinc}^2\left(\frac{\pi}{2} x\right) \, [\, p(1-p) \; + p^2 \delta(x) \; + p^2 \sum_{m \text{ odd}} \delta(x - m) \,] \qquad (36.9)$$

and with p = 1/2 this becomes [recall $x = f T_1$]

$$\langle \mathcal{P}(\omega) \rangle \; = \; (V/2)^2 \, (1/\omega_1) \, \mathrm{sinc}^2\left(\frac{\pi}{2} x\right) \, [\, 1/4 \; + 1/4 \, \delta(x) \; + 1/4 \sum_{m \text{ odd}} \delta(x - m) \,] \qquad (36.9)$$

Now write

$\delta(x) = \delta(f T_1) = (1/T_1)\delta(f)$
$\delta(x-m) = \delta(f T_1 - m) = \; (1/T_1)\delta(f - m/T_1)$

so my result can be written

$$\langle P(f) \rangle / 2\pi = (V/2)^2 \, (T_1/2\pi) \, \mathrm{sinc}^2\left(\frac{\pi}{2} f T_1\right) \, [\, 1/4 \; + 1/4 \, (1/T_1)\delta(f) \; + 1/4 \, (1/T_1) \sum_{m \text{ odd}} \delta(f - m/T_1) \,]$$

Now I have to use, for m odd,

$$\mathrm{sinc}^2\left(\frac{\pi}{2} m\right) \; = \sin^2(\pi m/2)/(\pi m/2)^2 = (2/\pi m)^2$$

and then

$$\langle P(f)\rangle = (V/2)^2\, T_1 \left\{ 1/4\, \mathrm{sinc}^2\left(\frac{\pi}{2} f T_1\right) + 1/4\, (1/T_1)\delta(f) + (2/\pi m)^2\, 1/4\, (1/T_1) \sum_{m\ \mathrm{odd}} \delta(f - m/T_1) \right\}$$

Xiong's unipolar RZ seems to be the same as my waveform at least, But here is his result

$$\Psi_s(f) = \frac{A^2 T}{16} \left(\frac{\sin \pi f T/2}{\pi f T/2} \right)^2 \left[1 + R_b \sum_{k=-\infty}^{\infty} \delta(f - n R_b) \right] . \quad \text{(unipolar RZ)} \tag{2.31}$$

but what is R_b ? Earlier he says it is 1/T. Well, his form does not show explicitly that the even lines are missing. Let's start over with my OTHER form where I have

$$[\, p(1-p) + p^2 \sum_{m=-\infty}^{\infty} \delta(x - m)\,] = [1/4 + 1/4 \sum_{m=-\infty}^{\infty} \delta(x - m)\,] = [\, 1/4 + 1/4\, (1/T_1) \sum_{m=-\infty}^{\infty} \delta(f - m/T_1)\,]$$

Then my expression says

$$\langle P(f)\rangle/2\pi = (V/2)^2\, (T_1/2\pi)\, \mathrm{sinc}^2\left(\frac{\pi}{2} f T_1\right) [\, 1/4 + 1/4\, (1/T_1) \sum_{m=-\infty}^{\infty} \delta(f - m/T_1)\,]$$

$$\langle P(f)\rangle = (V^2/16)\, T_1\, \mathrm{sinc}^2\left(\frac{\pi}{2} f T_1\right) [\, 1 + (1/T_1) \sum_{m=-\infty}^{\infty} \delta(f - m/T_1)\,]$$

and this DOES agree with Xiong's result. Fine. Verification complete.

2. Manchester Line Code

My result here is

$$\langle \mathcal{P}(\omega)\rangle = (V/2)^2\, (1/\omega_1)\, \sin^2\left(\frac{\pi}{2} x\right) \mathrm{sinc}^2\left(\frac{\pi}{2} x\right) [4p(1-p) + (2p-1)^2 \sum_{m = \pm \mathrm{odd}} \delta(x - m)\,]$$

which with p = 1/2 becomes

$$\langle \mathcal{P}(\omega)\rangle = (V/2)^2\, (1/\omega_1)\, \sin^2\left(\frac{\pi}{2} x\right) \mathrm{sinc}^2\left(\frac{\pi}{2} x\right)$$

$$\langle P(f)\rangle/2\pi = (V/2)^2\, T_1/2\pi\, \sin^2\left(\frac{\pi}{2} x\right) \mathrm{sinc}^2\left(\frac{\pi}{2} x\right)$$

$\langle P(f) \rangle = (V/2)^2 T_1 \sin^2(\frac{\pi}{2} x) \operatorname{sinc}^2(\frac{\pi}{2} x)$

Here is the Xiong result for Manchester, where $x = fT_1$

$$\Psi_s(f) = A^2T \left(\frac{\sin \pi fT/2}{\pi fT/2} \right)^2 \sin^2 \pi fT/2. \qquad \text{(Bi-}\Phi\text{-L)} \qquad (2.38)$$

His result seems to be 4 times larger than mine!

Yes, I found a bug in my work starting near 36.11, am fixing it now. Fix is A/2 = V. I did a complete repair just now. My result got 4x larger and now matches Xiong, thank you Mr. X!

So I have now verified Manchester with Xiong for p = 1/2.

Question: How does Xiong handle negative frequency in his results?

He says absolutely nothing about it! I just have to assume that his spectrum is for f in (-∞.∞) which is the way my stuff works and we are in agreement with the above examples./

3. AMI Code

My pulse is height V, and my result is this

$$\langle \mathcal{P}(\omega) \rangle = \mathcal{P}_{\mathtt{pulse}}(\omega) \frac{(1-a^2)\,[\sin^2(\pi x)]}{1+a^2-2a\cos(2\pi x)} \qquad a = (1-2p)\ . \qquad (37.11)$$

I go on to say that with p = 1/2 the result is

$$\langle \mathcal{P}(\omega) \rangle = \mathcal{P}_{\mathtt{pulse}}(\omega)\ \sin^2(\pi x) \qquad p = 1/2 \qquad (37.12)$$

$$\langle \mathcal{P}(\omega) \rangle = V^2\ (1/\omega_1) \operatorname{sinc}^2(\pi x) \sin^2(\pi x) \quad p = 1/2 \qquad x = \omega/\omega_1 \qquad (37.13)$$

Then my result says that

$\langle P(f) \rangle / 2\pi = V^2\ T_1/2\pi \operatorname{sinc}^2(\pi x) \sin^2(\pi x) \quad x = fT_1$

$\langle P(f) \rangle = V^2\ T_1 \operatorname{sinc}^2(\pi fT_1) \sin^2(\pi fT_1)$

Xiong approaches this subject on page 35. He calls AMI the name AMI-NRZ pseudoternary since it does in fact have 3 levels. Eventually he quotes this result

$$\Psi_s(f) = A^2T \left(\frac{\sin \pi fT}{\pi fT} \right)^2 \sin^2 \pi fT. \qquad \text{(AMI-NRZ)} \qquad (2.34)$$

and we have verification for this difficult case as well!! Hurray!

Still to go: NRZI and hold/change linecodes.

On 5.13.13 I integrated all these Xiong verifications into spectral doc, and I comment more on the Bennet Davey verification and its factor of 2 and I maintain their credit for the whole discussion.