Note that page numbering is turned on in this template.

Example to put after (34.11)

y_m = A if m is even, = B if m is odd

$$X''(z) = X'(\omega)/T_1 = \sum_{n=-\infty}^{\infty} y_n e^{-i\omega n T_1} = A \sum_{n \text{ even}} e^{-i\omega n T_1} + B \sum_{n \text{ odd}} e^{-i\omega n T_1}$$

$$n = 2m \qquad n = 2m+1$$

$$= A \sum_{m=-\infty}^{\infty} e^{-i\omega(2m)T_1} + B \sum_{m=-\infty}^{\infty} e^{-i\omega(2m+1)T_1}$$

$$= (A + Be^{-i\omega T_1}) \sum_{m=-\infty}^{\infty} e^{-i\omega(2m)T_1}$$

$$= (A + Be^{-i\omega T_1}) \sum_{m=-\infty}^{\infty} 2\pi\delta(2\omega T_1 - 2\pi m) \qquad \text{// using (13.2)}$$

$$= (1/2)(A + Be^{-i\omega T_1}) \sum_{m=-\infty}^{\infty} 2\pi\delta(\omega T_1 - \pi m) \qquad \text{// using (13.2)}$$

Thus

$$X''(z) = (A + Be^{-i\omega T_1}) \sum_{m=-\infty}^{\infty} 2\pi\delta(2\omega T_1 - 2\pi m)$$

$$|X''(z)|^2 = |A + Be^{-i\omega T_1}|^2 [2\pi\delta(0)] \sum_{m=-\infty}^{\infty} 2\pi\delta(2\omega T_1 - 2\pi m)$$

$$|X''(z)|^2 = (1/4)|A + Be^{-i\omega T_1}|^2 [2\pi\delta(0)] \sum_{m=-\infty}^{\infty} 2\pi\delta(\omega T_1 - \pi m)$$

$$= (1/2)|A + Be^{-i\omega T_1}|^2 [2\pi\delta(0)] \sum_{m=-\infty}^{\infty} 2\pi\delta(2\omega T_1 - 2\pi m)$$

$$|X(\omega)|^2 = |X_{pulse}(\omega)|^2 \; |X''(z)|^2$$

$$\frac{|X(\omega)|^2}{[2\pi\delta(0)]} = |X_{\mathbf{pulse}}(\omega)|^2 \; |A + Be^{-i\omega T_1}|^2 \sum_{m=-\infty}^{\infty} 2\pi\delta(2\omega T_1 - 2\pi m)$$

$$\mathcal{P}(\omega) = \frac{|X(\omega)|^2}{2\pi T} = \frac{|X(\omega)|^2}{2\pi T_1[2\pi\delta(0)]}$$

$$= |X_{\mathbf{pulse}}(\omega)|^2 \, (1/T_1) \; |A + Be^{-i\omega T_1}|^2 \sum_{m=-\infty}^{\infty} \delta(2\omega T_1 - 2\pi m)$$

$|A + Be^{-i\omega T_1}|^2 = (A + Be^{-i\omega T_1})(A + Be^{+i\omega T_1}) = A^2 + B^2 + 2AB\cos(\omega T_1)$

$\delta(2\omega T_1 - 2\pi m) = (1/2T_1)\, \delta(\omega - m(\omega_1/2))$

$$\mathcal{P}(\omega) = (1/2) \, |X_{\mathbf{pulse}}(\omega)|^2 \, (1/T_1)^2[A^2 + B^2 + 2AB\cos(\omega_1 T_1)] \sum_{m=-\infty}^{\infty} \delta(\omega - m(\omega_1/2))$$

Special case class: Suppose $x_{\mathbf{pulse}}(t)$ = box height 1 width T_1.

$$X_{\mathbf{pulse}}(\omega) = X(\omega) = T_1 \, \mathrm{sinc}(\omega T_1/2)$$

$$\mathcal{P}(\omega) = (1/2) \, \mathrm{sinc}(\omega T_1/2) \, [A^2 + B^2 + 2AB\cos(\omega_1 T_1)] \sum_{m=-\infty}^{\infty} \delta(\omega - m(\omega_1/2))$$

Special SubCase #1. A = 1 and B = 1.

$$[1 + 1 + 2\cos(\omega_1 T)] = 4\cos^2(\omega T_1/2)$$

$$\mathcal{P}(\omega) = (1/2) \, \mathrm{sinc}(\omega T_1/2) \, 4\cos^2(\omega T_1/2) \sum_{m=-\infty}^{\infty} \delta(\omega - m(\omega_1/2))$$

$$= 2 \sum_{m=-\infty}^{\infty} \mathrm{sinc}(m\pi/2) \cos^2(m\pi/2)\delta(\omega - m(\omega_1/2))$$

The factor here contains for $m \neq 0$

$$2 \sin(m\pi/2) \cos(m\pi/2) = \sin(m\pi) = 0$$

so only the m = 0 term survives and the result is

$$\mathcal{P}(\omega) = 2 \, \delta(\omega)$$

But why do I get 2 here? Does my δ squaring operation fail?

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I need more work on when you can do the $2\pi\delta(0)$ thing with a clear explanation of what scaling is allowed and what is not! I have swept this under the rug until now. More work needed as usual.