New Section 36 first part only PhL 3.26.13

What role has the autocorrelation function played in our development?

In Section 32 the autocorrelation function was first introduced, as $r_x(t)$ for x(t), and was computed for the simple case of a box pulse. Treating the definition of $r_x(t)$ as a convolution equation, the Wiener-Khintchine Relation $R_x(\omega) = |X(\omega)|^2$ was trivially derived. It was then shown that the spectral energy density of a pulse train $\mathcal{E}(\omega) = (1/2\pi)\, R_x(\omega)$ due to this relation. Finally, it was shown that $r_x(0) = E$, the total energy in the pulse train. We then commented on the origin of the name, showing that the auto-correlation function is the cross-correlation function of a function with itself when that function is real.

In Section 33 the autocorrelation function went unmentioned until the very end where it was again rather trivially noted that $P = r_x(0)/T$ since P = E/T, and that $\mathcal{P}(\omega) = R_x(\omega)/(2\pi T)$ since $R_x(\omega) = |X(\omega)|^2$.

In Sections 34 and 35 no reference was made at all to the autocorrelation function.

This leads us to make several comments:

(1) The autocorrelation function played no role whatsoever in our development of key equations such as

$$\langle \mathcal{P}(\omega) \rangle = \mathcal{P}_{pulse}(\omega) \left[(\beta-\alpha) + \alpha \sum_{m=-\infty}^{\infty} 2\pi\, \delta(\omega T_1 - 2\pi m) \right] \qquad \text{// infinite} \qquad (35.7)$$

(2) In some textbooks, one gets the impression that the autocorrelation function is somehow crucial for the development of such equations. It is not.

(3) Nevertheless, since $\mathcal{P}(\omega) = R_x(\omega)/(2\pi T)$, one can start with a description of $x^{(i)}(t)$ of pulse train i in a statistical ensemble, compute from it $r_x^{(i)}(t)$ and from that $R_x^{(i)}(\omega)$. One could then do a statistical average to obtain $\langle \mathcal{P}(\omega) \rangle = (2\pi T)^{-1}(1/I) \Sigma_{i=1}^{I} R_x^{(i)}(\omega)$. So it is possible to take a pathway to deriving equations like (35.7) which does pass through the land of the autocorrelation function.

(4) Our main reason for even bringing it up is that the autocorrelation is closely related to what we are doing, and in other applications such as those involving noise (and pseudo-noise PN) it becomes more significant. In such applications, the definition of $r_x(t)$ might include a normalizing factor so it is then autocorrelation per unit time, or per pulse (per chip in the PN world).