

FYSP102 / K2. MODULUS OF ELASTICITY

Purposes of the work:

- To become familiar with properties of solid materials in more details than during the lectures
- To rehearse performing repeated calculations with a PC

Due to limited time available in basic courses, properties of materials are touched only very shortly. Therefore in this work, you'll get acquainted with the modulus of elasticity for bending- a phenomenon which is more important in the education of an engineer than that for a physicist.

In the data analysis calculations using same formulas are repeated many times, therefore a computer is a useful tool. This work gives you a good opportunity to rehearse how to use Excel for repeated calculation of quantities.

General

Elasticity properties of materials are discussed in the text book Young & Freedman (11th ed.), sc. 11.4 and 11.5.

The home work related to this laboratory work is in the end theory section. The home work should be worked out before measurements.

1. Theory

1.1. Hooke's law

Solid bodies tend to conserve their shape and volume. Under induced stress they behave differently compared to gases and liquids. A simple example is a solid rod (length L , cross section A) fixed from one end and an external, axial force F acting on the other end. The

force F acts equally on every cross section A independent of its position. The pulling tension in each position is F/A .

The tension F/A , depending of its direction either stretches or compresses the rod. The amount of relative extension $\Delta L/L$ obeys in linear approximation Hooke's law

$$\frac{F}{A} = E \frac{\Delta L}{L}, \quad (1)$$

where E is the modulus of elasticity of the material the rod is made of. The linear Hooke's law is valid for small, elastic and reversible extensions, only.

1.2. Modulus of elasticity for bending

Forces needed to bend (a rod) are usually much smaller than those needed in stretching. Unfortunately the theory for bending is more cumbersome. Let's imagine that the other end of a rod is firmly fixed and the other end is loaded with some weight, the rod bends. Various layers of the rod bend differently depending of their position. In the middle there is a neutral layer which does neither stretch nor compress. Layers situated above or underneath either stretch or compress. The neutral layer is located at the centre of mass of a cross section A .

In fig. 1 a rod is being bent. The distance between the stretched layer (DD') and the neutral layer (CC') is denoted with s , the radius of curvature with R and the angle of curvature with ϕ . From the figure it can be seen that the length CC' is $R\phi$ and that of DD' is $(R+s)\phi$. The relative stretching of DD' compared to CC' is thus $s\phi/R\phi = s/R$. Hooke's law gives now

$$\frac{dF}{dA} = E \frac{s}{R}, \quad (2)$$

where dF is the infinitesimal force tending to reverse an infinitesimal layer dA to its original, equilibrium length. Notice that the external, bending force acts not along the rod,

but with a relative stretching $\Delta L/L$ achieved, the internal stretching force F/A obeys Hooke's law.

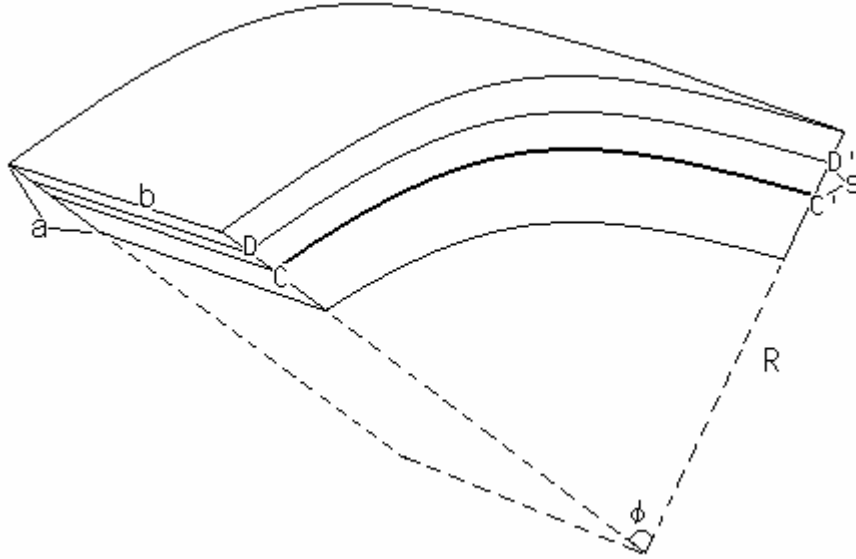


Figure 1. A bending rod (thickness a , width b)

1.3. Surface momentum of inertia

In static equilibrium the torques of external forces and the internal tensions cancel each other. The infinitesimal force dF causes a torque $dM = s dF$ to the cross section dA . Eq. (2) gives now

$$dF s = E \frac{s^2}{R} dA \quad (3)$$

The total torque M_S is thus

$$M_S = \int E \frac{s^2}{R} dA = \frac{E}{R} \int s^2 dA = \frac{E}{R} I_A \quad (4)$$

I_A is called as the surface momentum of inertia or the square-momentum of inertia of the surface. I_A depends of the shape of A and of the axis respect to which it is calculated. For a cross section of rectangular shape (fig. 2)

$$I_A = \int_{-a/2}^{a/2} s^2 dA = \int_{-a/2}^{a/2} s^2 b ds = \frac{b}{3} \left[s^3 \right]_{-a/2}^{a/2} = \frac{ba^3}{12}, \quad (5)$$

and for a cylindrical rod (derive!)

$$I_A = \frac{\pi r^4}{4} = \frac{\pi d^4}{64}, \quad (6)$$

where r is the radius and d the diameter of the solid cylinder.

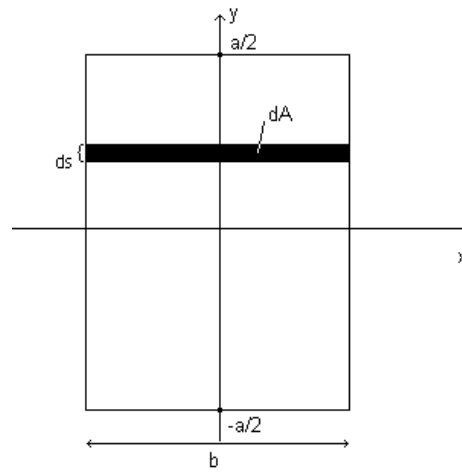


Figure 2. Cross section of a rectangular rod.

1.4. Radius of Curvature

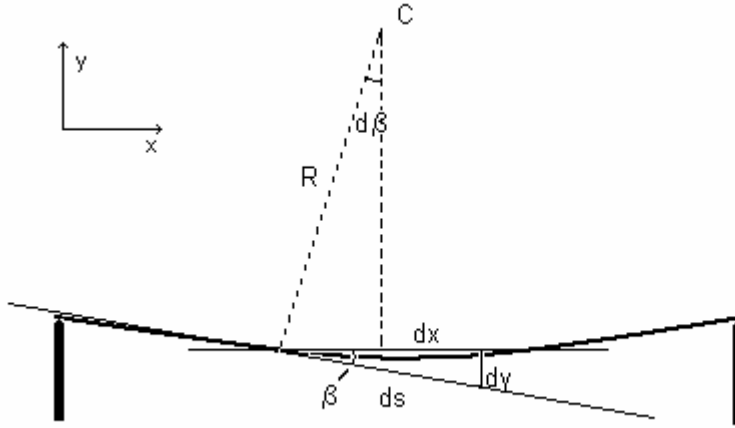


Figure 3. A bent rod and radius of curvature R

In order to calculate the modulus of elasticity E one needs to know the radius of curvature. For small displacements in x-y co-ordinates $1/R \approx d^2y/dx^2$ (see Pennala 1995). Eq. 4 reads now

$$M_s = EI_A \frac{d^2y}{dx^2}. \quad (7)$$

In the experiment you'll use rods suspended from both ends and bend with weights placed in middle (the origin for calculations). A force F in origin is compensated by suspending forces $F/2$ in the ends. The torque of the suspending forces must equal to the torque M_u of the internal tension (forces):

$$EI_A \frac{d^2y}{dx^2} = \frac{F}{2} \left(\frac{l}{2} - x \right), \quad (8)$$

where l is the length of the rod. By separating variables x and y and integrating twice (do this!) you'll get

$$EI_A y = \frac{F}{2} \left(\frac{lx^2}{4} - \frac{x^3}{6} \right), \quad (9)$$

from which function $y(x)$ is

$$y(x) = \frac{F}{2EI_A} \left(\frac{lx^2}{4} - \frac{x^3}{6} \right) \quad (10)$$

At both support positions $x = l / 2$ i.e.

$$y = \frac{Fl^3}{48EI_A} \quad (11)$$

This result gives the displacement at the middle of the rod when the force F acts to the middle point, when the distance between the suspension points is l .

Home work:

- How much does a metal rod, suspended from two points, bend, if the middle point is loaded with 1,0 kg. The distance between suspension points is 0,50 m and the cross section is of rectangular shape ($b = 1,0$ cm and $a = 0,50$ cm, see figure 1). Compute the displacement of the middle point for aluminium and steel rods.
- Do the rods bend more or less if values of quantities a and b are interchanged?
- Which one bends more easily, a rod whose cross section is rectangular (each side equal to s) or a rod with circular cross section (diameter equal to s)? The rods are made of same material.

2. Equipment and measurements

Metal rods to be studied are suspended with two stands placed on the ends. Use several weights in the middle and measure the amount of bend in the middle with a micrometer screw. Measure the dimensions of the rods carefully. Use at least three different rods made of two different materials and having different geometries (rectangular and cylindrical).

4. Analysis of results

Plot amount of bending Δy as a function of the load (force applied). Using the method of least square sum define the linear coefficients (with error) of the straight lines fitting best to the measured points. In each case, calculate from the slope the modulus of elasticity.

Compare the measured values of E to values in literature.

Error estimate for E can be obtained easily using the max-min-method.

Literature:

Pennala, Erkki: "Lujuusopin perusteet". Helsinki, Otatieto OY 1995, ch. 4, p.'s. 91-