Potential Problem Analogies PhL 8.5.10

This is very incomplete. Use split-screen to compare two worlds.

1. Electrostatics:

potential V

potential difference ΔV drives electric current dQ/dt via ohms law

current $I = dQ/dt = \Delta V / R$

surface of metal is place where V = constant

electric field $\mathbf{E} = -\nabla V$ = electric flux density, constant = 1

current density $\mathbf{J} = \kappa \mathbf{E}$ constant = κ = electric conductivity

$\partial_{\mathbf{n}} V = E_{\mathbf{n}} = 4\pi\sigma$ at a metal boundary surface, σ = charge density = electric flux source

$\nabla^2 V \sim \rho$ = 3D charge density

$\partial_{\mathbf{n}} V \sim \sigma$ = charge density source

Dirichlet BC: specify V on boundary

Neumann BC: charge density flow on boundary

positive and negative charge

charge-free boundary (such as cap of bowl) has $\partial_{\mathbf{n}} V = 0$

2. Heat flow:

temperature u

temperature difference Δu drives heat current dQ/dt via the conduction heat transfer law

heat current $I = dQ/dt = \Delta u / R$ R = thermal resistor resistance

surface against heat reservoir is place where u = constant

heat flow field $\mathbf{F} = -k \nabla u(\mathbf{x})$ constant = k = thermal conductivity

heat current density $\mathbf{J} = \mathbf{F}$ constant = 1

$\partial_n u$ = heat current source at a boundary = (-1/k) $\mathbf{F}_n$ = like charge density σ but has no name

(heat flow: $\partial_t Q = -k\,\partial_n u$ $\partial_t Q$ is like σ, heat flow through unit area of surface.)

$\nabla^2 u$ = 3D heat source density = q(x)

$\partial_t u - (k/c)\nabla^2 u = f(x,t)/c$ = 3D heat source // Stak v I p 328

static temperature distribution: $-(k/c)\nabla^2 u = f(x,t)/c$ $f \sim \rho$

$-k\nabla^2 u = f(x,t)$

dimensions: f = energy/vol/time. k = energy/time/degree TBC

Dirichlet BC: specify u on boundary

Neumann BC: specify heat flow $\partial_n u$ on boundary

positive and negative heat flow source (source and sink)

insulated boundary means $\partial_n u = 0$ since no heat flow cross boundary. (Neumann)

3. Fluid flow

velocity field **v** is like **E**

$\mathbf{v} = \nabla\varphi$ φ is the velocity potential

hard boundary wall sets $\mathbf{v}_{perp} = 0 = \partial_n\varphi$ = Neumann with σ = 0

$\partial_n\varphi \sim$ source at a boundary

also at wall we have $\mathbf{v}_{parallel} = 0$ like we have $\mathbf{E}_{parallel} = 0$ at a metal wall.
(so this is true where the wall has no source or sink, ie, it is not a leaky wall)