Collection of Distributional Facts PhL 6.18.09

1. Delta sequence: $s_k(x) = (1/\pi)\, k/(1 + k^2x^2)$ $\lim_{k\to\infty} s_k(x) = \delta(x)$ // I p 19A

2. Delta sequence: $s_k(x) = \sin(kx)/(\pi x)$ $\lim_{k\to\infty} s_k(x) = \delta(x)$ // I p 27 A

3. Test function in S_1: $\varphi(x) = \exp(1/[x^2-1])$ $|x| < 1$ // I p 29 A

4. Fact: $\delta(x) = (1/2\pi) \int dy\, e^{iyx}$ // I p 43 B

5. $g_n(x) = -\cos(nx)/n$ $\lim_{n\to\infty} g_n(x) = 0$ // I p 44 B

6. $\lim_{n\to\infty} \sin(nx) = 0$ // I p 44 B

7. Fact: $\Sigma_{int\ k}\, \delta(x-k) = 1 + 2\, \Sigma_{n=1}^{\infty} \cos(2n\pi x)$ // I p 45 1.22 and similar facts

8. $d/dx\ \log(|x|) = pf(1/x)$ $< pf(1/x), \varphi> = \fint (1/x)\, \varphi(x)$ // I p 49

9. $\lim_{R\to\infty}\ (1-\cos(Rx))/x = pf(1/x)$ // I p 49 E

10. $\lim_{\alpha\to\pm 0}\ 1/(x+i\alpha) = \mp\, i\pi\delta(x) + pf(1/x)$ // I p 50 1.27

11. Translation rule: $< t(x-a), \varphi(x)> = < t(x), \varphi(x+a)>$ // I p 35

12. Scaling rule: $< t(ax), \varphi(x)> = (1/|a|) < t(x), \varphi(x/a)>$ // I p 36

13. Derivative rule: $<t^{(n)}(x), \varphi(x)> = (-1)^n <t(x), \varphi^{(n)}(x)>$ // I p 37 1.19a

14. Fact: $\lim_{R\to\infty} (1/\pi) \sin^2(Rx)/(Rx^2) = \delta(x)$ // II p 16 5.8