Hi Reyer Sjamaar, Aug 22, 2015

I am a physics guy (unfortunately older than you) who was interested in learning something about differential forms and manifolds. I luckily found your excellent notes, downloaded them, printed them, and studied them pretty hard, marking them up as I went (though I admit, I only did a very few problems). I found your presentation *very* well written and well attuned to my ability to understand things. As for many physics people, dense math writing (where words are avoided wherever possible) is like castor oil in print (credit: Mark Twain said the Book of Mormon was chloroform in print). I was always able to use the web to clarify confusions and to identify terms which did not appear in your document (e.g. "cotangent space" being dual to the tangent space).

It was just my luck to download your Aug 2006 version on June 28 of this year, and yesterday after finishing reading the whole thing I noticed your new July 2015 update, the first in 9 years. I see that you have made many stylistic and content alterations, no doubt all for the better. Hey, the graphs are even clipped to their allotted areas (but I thought the overflowing graphs were a bit artsy).

As I was reading your 2006 version, I took note of a few errata with the intention of reporting them to you. I maintain my own set of documents online and it is an endless battle to iron out errata, and when I update the documents, new errata sometimes appear, so I always appreciate it when people report even small errata to me. It is the beauty of the online document -- always updateable!

You fixed most of my errata, so I have only three tiny typo-class items to report:

- page 22 top line: reads "for al" should be "for all" (page 21 of old PDF)

- page 42 equation (3.2) : $\Sigma_{\mathtt{I}}$ is missing on the middle expression (page 40 of old PDF)

- page 80 third line of (6.2): "can be hard find" should be "can be hard to find" (page 72 of old PDF)

It was interesting to see how you dealt with issues I have to deal with. One example is writing column vectors horizontally, which you now do in your new PDF without adding a $^{\mathtt{T}}$ superscript. Everything is a tradeoff, the reader just has to understand.

It was probably good that you gave up on Gausβ since most Ithaca students likely don't speak German, but we still have the endless question of possessives. In my life it was always Gauss's Theorem so to be consistent I have to say Stokes's Theorem which sounds terrible. The web has much on this topic with various obscure standards. For you it is Gauss' Theorem and Stokes' Theorem. Another term in my history is "the Green's function" of a differential operator, which friend J.D. Jackson says we how have to refer to as "the Green function". It hurts to say it. And "the Maxwell equations", not Maxwell's equations. Oy.

Another issue is "equation numbers" and you are somewhat of a minimalist in this area. Should figures have "equation numbers" or a different system of "figure numbers" or no numbers at all? How do you refer to an equation or figure that has no reference? I notice that you have moved into the brave new world of clickable equation numbers. And is it "2.3 Example" or "Example 2.3" ? On it goes.

The wedge product and the sort of parallel universe between the alternating functional Grassmann stuff and "physics stuff" needs more pondering on *my* part. Us peons feel uncomfortable about $dx_i dx_j = - dx_j dx_i$ without some operator symbol in there, but lots of wedge operators certainly do clutter things up. But having no ∧ symbols sometimes leads to ambiguity. For example, in your critical proof of Stokes' Theorem on page 70 (new PDF) you have

$$\int_c d\alpha = \sum_{i=1}^{k} \int_{[0,1]^k} d(g_i\, dt_1\, dt_2 \cdots \widehat{dt}_i \cdots dt_k) = \sum_{i=1}^{k} (-1)^{i+1} \int_{[0,1]^k} \frac{\partial g_i}{\partial t_i}\, dt_1\, dt_2 \cdots dt_k.$$

Changing the order of integration (see Remark 5.3) and subsequently applying the fundamental theorem of calculus in one variable, formula (B.1), gives

$$\begin{aligned}\int_{[0,1]^k} \frac{\partial g_i}{\partial t_i}\, dt_1\, dt_2 \cdots dt_k &= \int_{[0,1]^k} \frac{\partial g_i}{\partial t_i}\, dt_i\, dt_1\, dt_2 \cdots \widehat{dt}_i \cdots dt_k \\ &= \int_{[0,1]^{k-1}} (g_i(t_1, \ldots, t_{i-1}, 1, t_{i+1}, \ldots, t_k) \\ &\qquad - g_i(t_1, \ldots, t_{i-1}, 0, t_{i+1}, \ldots, t_k))\, dt_1\, dt_2 \cdots \widehat{dt}_i \cdots dt_k.\end{aligned}$$

In the first line (for $\int_c d\alpha$) the implied detail work is this (red means something missing)

$d(g_i dt_1 dt_2 ... dt_i ... dt_k) = [d(g_i)] dt_1 dt_2 ... dt_i ... dt_k = [\Sigma_j (\partial_j g_i) dt_j] dt_1 dt_2 ... dt_i ... dt_k$

$= [\partial_i g_i) dt_i] dt_1 dt_2 ... dt_i ... dt_k \;\; = (-1)^{i+1} dt_1 dt_2 ... dt_i ... dt_k \;\; = (-1)^{i+1} dt_1 dt_2 .... dt_k$

so wedge symbols are implicitly present here since the slide of dt_i creates $(-1)^{i-1} = (-1)^{i+1}$. However, in the next line the dt_i is slid back to the left with no incurred sign, so here it seems the wedge symbols are not implied.

I wonder how many people have downloaded and studied your PDF? If in the hundreds, then perhaps most of your teaching effectiveness is through that PDF and not in the classroom. Maybe your web site has a counter for such things. And if you don't officially publish your PDF, I hope you have a way to make it persist "in perpetuity", something of interest to me (like in the arXiv but they probably don't accept "expository documents").

Anyway, thanks again for writing a wonderful document. I really appreciate the effort you expended to get it right, and I'm sure your classroom students do as well!

Best regards.

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P.S. I sort of assume it is Sjamaar as in " cha-mar' " ? Maybe help me out on that.
As you see, my last name is Dutch, but I cannot speak that language.