Wiki on Exterior Algebra PhL 7.16.15

Let's try hacking into this just for fun. Here is the opening paragraph.

In mathematics, the **exterior product** or **wedge product** of vectors is an algebraic construction used in Euclidean geometry to study areas, volumes, and their higher-dimensional analogs. The exterior product of two vectors u and v, denoted by $u \wedge v$, is called a bivector and lives in a space called the *exterior square*, a geometrical vector space that differs from the original space of vectors. The magnitude[3] of $u \wedge v$ can be interpreted as the area of the parallelogram with sides u and v, which in three dimensions can also be computed using the cross product of the two vectors. Also like the cross product, the exterior product is anticommutative, meaning that $u \wedge v = -(v \wedge u)$ for all vectors u and v. One way to visualize a bivector is as a family of parallelograms all lying in the same plane, having the same area, and with the same orientation of their boundaries—a choice of clockwise or counterclockwise.

When regarded in this manner the exterior product of two vectors is called a 2-blade. More generally, the exterior product of any number k of vectors can be defined and is sometimes called a k-blade. It lives in a geometrical space known as the k-th exterior power. The magnitude of the resulting k-blade is the volume of the k-dimensional parallelotope whose sides are the given vectors, just as the magnitude of the scalar triple product of vectors in three dimensions gives the volume of the parallelepiped spanned by those vectors.

So what things does this say?

1. u ^ v is a product of two **vectors,** and as such is a **bivector** or a **2-vector**.

2. Bivectors live in some space of bivectors called the **exterior square**.

3. **Magnitude** of the bivector u ^ v is the area of the parallelogram spanned by the two vectors.

4. Above suggests that the place where the edge vectors of the 2-piped meet, normally at the **origin**, could be at any point in the plane you want, hence "**family** of 2-pipes in a plane", all aligned the same, just translated.

5. the wedge product u ^ v is sometimes called a **2-blade.**

5a. I presume that u ^ v ^ w would be called a **3-blade**. and so we know then what a **k-blade** is.

6. A fancy word for an n-piped is an n-**parallelotope**.

Lets try to swallow another chunk of this wiki page:

The **exterior algebra**, or **Grassmann algebra** after Hermann Grassmann,[4] is the algebraic system whose product is the exterior product. The exterior algebra provides an algebraic setting in which to answer geometric questions. For instance, whereas blades have a concrete geometrical interpretation,

objects in the exterior algebra can be manipulated according to a set of unambiguous rules. The exterior algebra contains objects that are not just *k*-blades, but sums of *k*-blades; such a sum is called a *k*-vector.[5] The *k*-blades, because they are simple products of vectors, are called the simple elements of the algebra. The *rank* of any *k*-vector is defined to be the smallest number of simple elements of which it is a sum. The exterior product extends to the full exterior algebra, so that it makes sense to multiply any two elements of the algebra. Equipped with this product, the exterior algebra is an associative algebra, which means that $\alpha \wedge (\beta \wedge \gamma) = (\alpha \wedge \beta) \wedge \gamma$ for any elements α, β, γ. The *k*-vectors have degree *k*, meaning that they are sums of products of *k* vectors. When elements of different degrees are multiplied, the degrees add like multiplication of polynomials. This means that the exterior algebra is a graded algebra.

7. The idea that a sum of 2-blades is a 2-vector seems very strange. I guess each blade is a 2-vector, and there is some vector space that let's you add 2-vectors, perhaps linear combinations. The two here is called the degree. So a k-vector has **degree** k. I don't see the difference between a 2-vector and 2-blade. Maybe the 2-blades form a basis then you linearly combine them to get an arbitrary 2-vector, just as you do with 1-vectors.

8. The ^ operation is associative.

Another gulp:

The definition of the exterior algebra makes sense for spaces not just of geometric vectors, but of other vector-like objects such as vector fields or functions. In full generality, the exterior algebra can be defined for modules over a commutative ring, and for other structures of interest in abstract algebra. It is one of these more general constructions where the exterior algebra finds one of its most important applications, where it appears as the algebra of differential forms that is fundamental in areas that use differential geometry. Differential forms are mathematical objects that represent infinitesimal areas of infinitesimal parallelograms (and higher-dimensional bodies), and so can be integrated over surfaces and higher dimensional manifolds in a way that generalizes the line integrals from calculus. The exterior algebra also has many algebraic properties that make it a convenient tool in algebra itself. The association of the exterior algebra to a vector space is a type of functor on vector spaces, which means that it is compatible in a certain way with linear transformations of vector spaces. The exterior algebra is one example of a bialgebra, meaning that its dual space also possesses a product, and this dual product is compatible with the exterior product. This dual algebra is precisely the algebra of alternating multilinear forms on *V*, and the pairing between the exterior algebra and its dual is given by the interior product.

Well nothing is digestible there, except the statement that there is a connection to differential forms.

That's enough for now!