I have run into this subject before, but not recently. I search my files and find only a reference to Clifford in my Ahlfors meta review concerning the "cross ratio". So I really have nothing at all on this subject.

I found this excellent and short PDF on the subject,

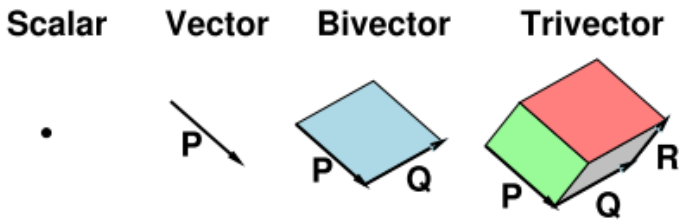
Introduction to Clifford Algebra

John Denker

This guy I think is a retired physics Prof from Cornell. His notes are 35 pages and seem relatively simple, so I will give them a shot, reading PDF on the screen.

2. Overview.

Here we have pictures of the "clifs" that inhabit this Clifford algebra.



object	visualized as	geometric extent	grade
scalar	point	no geometric extent	0
vector	line segment	extent in 1 direction	1
bivector	patch of surface	extent in 2 directions	2
trivector	piece of space	extent in 3 directions	3
etc.			

Different objects are classified by their "grade" as shown. Hence Clifford Algebra is a "graded algebra". He argues to get rid of cross products! We shall see.

2.3 Addition

Unlike normal algebras, you are allowed to add objects of different gradation. Normally we add vectors to get only vectors, but here you can add any of the "clifs" together. A clif is aka a "multivector". He shows graphically how you can add two 2-vectors together. Not totally obvious but OK for an example. Apples and oranges and fruit salad comment.

2.4 Grade Selection

Notation: The grade-N piece of C is denoted $\langle C \rangle_N$.

I guess given some mixed clif, you can project out whichever grade piece you want.

2.5 Multiplication: Preliminaries.

In order to have an "algebra" you need to know how to add elements, scale them by scalars, and how to multiply them. Adding clifs seems simple, as is scalar mult. But multiplying two clifs of possibly different grade takes some doing. The fact that you can do this is a major feature of the clif world.

2.6 Vectors times Vectors

For vector clifs (only) we get to say

P.Q = (1/2)[PQ+QP] dot
P∧Q = (1/2)[PQ-QP] wedge

PQ = P.Q + P∧Q // vectors only!

but we don't know yet what PQ means (known as a "geometric product")

As another corollary of equation 4, we see that the dot product of two vectors is symmetric, while the wedge product of two vectors is antisymmetric:

$$P \cdot Q = Q \cdot P \quad \text{where } P \text{ and } Q \text{ have grade} = 1 \tag{6a}$$
$$P \wedge Q = -Q \wedge P \quad \text{where } P \text{ and } Q \text{ have grade} \leq 1 \tag{6b}$$

Terminology: The wedge product is sometimes called the *exterior product*. (This is not to be confused with the tensor product $\otimes$, which is sometimes called the outer product.)

Exterior product $\neq$ outer product.

In this document we don't bother with the term exterior product, and just call it the wedge product.

Fine so far, but what does PQ mean? Well, so far this is only defined "axiomatically"

Axiom: We postulate that there is a *geometric product* operator that can be used to multiply *any* element of the Clifford algebra by any other.

Notation: The geometric product of A and B is written AB. That is, we simply juxtapose the multiplicands, without using any operator symbol.

We further postulate that the geometric product is associative and distributes over addition:

$$\begin{aligned} (AB)C &= A(BC) = ABC \\ A(B+C) &= AB + AC \end{aligned} \tag{3}$$

So whatever it is, it has to have these properties. Called a geometric product. Some are not so hard if one item is a scalar:

- As a special case of the geometric product, multiplying scalars by scalars is straightforward. This is just the familiar multiplication of real numbers. In this case, multiplication is commutative.
- As another special case, multiplying vectors by scalars is also straightforward, and is presumably familiar from ordinary vector algebra. This is another case where multiplication is commutative; that is, $sV = Vs$ for any scalar s and any vector V.
- For that matter, it is easy to multiply any clif by a scalar. This is always commutative; that is, $sC = Cs$ for any scalar s and any clif C.

2.7 About the dot product of vectors

The general idea is that this is pretty much like your physics normal dot product and it produces a scalar which is rotationally invariant. Fine.

2.8 Parallel and Perpendicular (vectors)

First, if you consider P ^ Q where P = Q, the result has to be zero since P^Q = -Q^P. Similarly, if the two vectors are collinear, so that P = αQ, you can show again that P ^ Q = 0. Similarly, if P = Q, we know that we have PQ = QP, and this should be true for collinear as well. That means that P.Q = 0. The Clifford word for collinear is "parallel" and one writes P || Q.

Perp or orthogonal is similar. If P and Q are perp, then P.Q = 0 and P^Q = PQ. This is all reminiscent of the regular dot product and cross product.

Define now:

$$P_Q := Q \frac{P \cdot Q}{Q \cdot Q}$$

What can we say about this new vector? Write

$P = P_Q + (P-P_Q)$

Notice that

$P_Q.Q = P.Q$
$P_Q \wedge Q = 0$ since $Q \wedge Q = 0$

$(P-P_Q).Q = 0$ trivial
$(P-P_Q) \wedge Q = P \wedge Q$

The first lines say that P_Q is parallel to Q.
The second lines say that $(P-P_Q)$ is perpendicular to Q.

So P = P_Q + (P-P_Q) decomposes a vector P into a part which is parallel to Q and a part that is perp to Q. This idea although perhaps interesting is not used anywhere below as far as I know.

2.9 Properties of Vector Wedge Products

Taking all these observations together, we find that $P \wedge Q$ behaves exactly as we would expect a bivector to behave, based on the description given in section 2.1: a patch of surface with a direction of circulation around its edge. That is: the area is unchanged if we rotate things in the plane of the surface, but if we rotate things 180 degrees in a plane perpendicular to the surface, the surface flips over, reversing the sense of circulation.

I think he means rotate 180 about an axis perp to the plane of the surface.

Claimed generalization is this:

$$P \wedge Q \wedge R := \frac{1}{6}(PQR + QRP + RPQ - RQP - QPR - PRQ) \tag{12}$$

$$q_1 \wedge q_2 \wedge q_3 \cdots q_r := \frac{1}{r!} \sum_{\pi} \text{sign}(\pi) q_{\pi(1)}\, q_{\pi(2)}\, q_{\pi(3)} \cdots q_{\pi(r)} \tag{13}$$

which agree with my Grassmann-based approach with functions instead of clifs.

Note added: In Spivak we talk about antisymmetrizing with respect to the arguments of a function. Here we are antisymmetrizing vectors of a Clifford algebra, where vectors are multiplied together by some method we don't at this point understand at all!

The claim is that the above two objects will be grade 3 and grade r. Somehow in a wedge product you are supposed to add the grades of the factors, reason not stated but perhaps reasonable.

The letters are all still vectors.

People like to say "the wedge product is antisymmetric" ... but you have to be careful. It is antisymmetric with respect to interchange of any two vectors ... not any two clifs. For example:

$$s \wedge C = C \wedge s \qquad \text{(not antisymmetric)} \tag{14}$$

for any scalar s and any clif C.

Fine by me. I think I found cases in the function world where this is also an issue (ie, no minus sign).

Terminology: A *blade* is defined to be any scalar, any vector, or the wedge product of any number of vectors.

So this is a wedge product of 0 or more vectors more or less.

Note added: I suppose a 2-vector and a 3-vector are examples of blades of grade 2 and 3. Here I am using Clif language for n-vector. Clif people usually write bivector or trivector to avoid this ambiguity!

Terminology: Any clif that has a definite grade is called *homogeneous*. It is necessarily either a blade or the sum of blades, all of the same grade.

OK, a sum of blades of the same grade makes a homogeneous clif.

Terminology: As previously mentioned, we use the term *clif* to cover the most general element of the Clifford algebra. In the literature, the same concept is called a *multivector*, but we avoid that term because it is misleading. The problem may be due in part to the etymology suggested by the sequence:

$$\text{vector, bivector, trivector, } \cdots \text{, multivector} \quad \text{(WRONG)} \tag{15}$$

in contrast to the correct sequence:

$$\text{vector, bivector, trivector, } \cdots \text{, blade} \quad \text{(RIGHT)} \tag{16}$$

OK good. You might think of all the items in the first list as multivectors of different grades, but the items in the first list are really blades. This seems to confirm my notion that an n-vector (Clif) is a blade.

Terminology: Do not confuse a trivector with a 3-vector. ok

Terminology: To describe the grade of a blade, the recommended approach is to mention the grade explicitly. For example, we say a bivector has grade=2, while a trivector has grade=3, and so on.

> There is another, non-recommended approach, in which a bivector is called a 2-blade, while a trivector is called a 3-blade, and so on. This is risky because of possible confusion as to whether the number refers to grade or dimension. Note that a 3-blade has grade=3, while a 3-vector has dimension=3, so confusion is to be expected.

Very good. There are two different integers to keep track up. So far so good.

2.10 Other Wedge Products

Above we wedged a vector with a vector. Here we generalize to blade with a blade, where the two blades can have different grades!

It is easy to see that for any two blades P and Q, which are of grade p and q respectively, the grade of $P \wedge Q$ will be $p + q$ (unless the product happens to be zero, in which case its grade is zero).

We can use that idea in the other direction, as follows: As discussed in section 2.14, the full geometric product PQ is liable to contain terms of all grades from $|p-q|$ to $p+q$ inclusive (counting by twos). The wedge product consists of just those terms with the highest possible grade. In symbols:

$$\begin{array}{lll} \text{If} & P & = \langle P \rangle_p \\ & Q & = \langle Q \rangle_q \\ \text{then} & P \wedge Q & := \langle PQ \rangle_{p+q} \end{array} \tag{18}$$

So this PQ discussion is where P and Q are blades.

So that claim then is that if you multiply two arbitrary clifs, the result will contain lots of graded components, and that component with the highest grade will be P^Q . Notice the notation above. I might want to prove this claim.

You can then consider clif ^ clif by breaking each into graded pieces so

$(\Sigma_i\ \alpha_i\ \text{Ablade}_i\ )\hat{}\ (\Sigma_j\ \beta_i\ \text{Bblade}_i\ ) = \Sigma_{ij}\alpha_i\beta_j\ \ (\text{Ablade}_i\ )\hat{}\ (\text{Bblade}_j\ )$

and to get $(\text{Ablade}_i\ )\hat{}\ (\text{Bblade}_j\ )$ we use the rule shown above = $\langle \text{Ablade}_i\text{Bblade}_j \rangle_{i+j}$

I get the general idea, perhaps need some examples.

Restate: P^Q has grade which is the sum of the two grades
PQ contains grades stepping down from this sum to | diff |, more complicated!

2.11 Other Dot Products

We hereby define the dot product of two blades to be the lowest-grade part of the geometric product. That is, if P has grade p and Q has grade q, then the dot product will have grade $|p-q|$. In symbols:

$$\begin{array}{lll} \text{If} & P & = \langle P \rangle_p \\ & Q & = \langle Q \rangle_q \\ \text{then} & P \cdot Q & := \langle PQ \rangle_{|p-q|} \end{array} \tag{20}$$

Just as the wedge product was the top-grade part of the geometric product, the dot product is the bottom-grade part.

This is done by fiat! It certainly works for 2 vectors where p=q=1 for grades, then |p-q| = 0, and in fact the entire thing P.Q in this case is in the bottom grade. It reminds me of representations of the rotation group.

Then

$(\Sigma_i\ \alpha_i\ \text{Ablade}_i\ )\bullet\ (\Sigma_j\ \beta_i\ \text{Bblade}_i\ ) = \Sigma_{ij}\alpha_i\beta_j\ \ (\text{Ablade}_i\ )\bullet\ (\text{Bblade}_j\ )$

and to get (Ablade$_i$)•(Bblade$_j$) we use the rule shown above = <Ablade$_i$Bblade$_j$>$_{|i-j|}$

So we are staying a bit axiomatic here, working first with blades, then with general clifs. We still have no idea how to multiply two arbitrary clifs to get PQ. Reader is getting impatient.

2.12 Wedge Product as Painting

Consider C ^ V where C is any clif and V is a vector clif. Draw clif C along V and let it "paint" as it goes, You then fill out the resulting Clif volume. I will let that ride, seems reasonable.

2.13 Chirality

The basic Clifford Algebra has no chirality (handedness). But you can add ingredients which add this chirality. This is part of Denker's promotional efforts.

> We make a point of keeping things non-chiral, to the extent possible, because it tells us something about the symmetry of the fundamental laws of physics, as discussed in reference [3].

Comments that the "Hodge dual" optional feature does have chirality.

2.14 More about the Geometric Product

This is what PQ is in all the above discussion. An example is given at the point of multiplying two blades each of grade 2.

Example: Let $\{\gamma_1, \gamma_2, \gamma_3, \gamma_4\}$ be a set of orthonormal spacelike vectors, as discussed in section 2.18, and define:

$$\begin{aligned} A &:= \gamma_1 \wedge \gamma_2 \\ B &:= (\gamma_2 + \gamma_3) \wedge (\gamma_4 + \gamma_1) \end{aligned} \tag{23}$$

Both A and B are homogeneous of grade 2. Indeed they are 2-blades. It is easy to calculate the geometric product:

$$AB = 1 + \gamma_1\gamma_4 + \gamma_2\gamma_3 + \gamma_1\,\gamma_2\,\gamma_3\,\gamma_4 \tag{24}$$

and therefore

$$\begin{aligned} A \cdot B &= \langle AB \rangle_0 = 1 && (25a) \\ &\langle AB \rangle_2 = \gamma_1\gamma_4 + \gamma_2\gamma_3 && (25b) \\ A \wedge B &= \langle AB \rangle_4 = \gamma_1\,\gamma_2\,\gamma_3\,\gamma_4 && (25c) \end{aligned}$$

We know that the 4 gammas is grade 4 only because we know it is A^B. Did I miss a rule that says the grade of a geometric product is the sum of the grades of the factors. That seems to be the case here. Wrong! Remember that the geometric product contains a whole series of grades, and we see that happening in the above example where AB = sum of three pieces of different grade.

The product of two vectors has A^B of grade 2 and A.B of grade 0. The grades always step by 2, unlike combining angular momenta! (Can I prove that?)

2.15 Spacelike, Timelike and Null

This relates only to vectors. V.V can be +, -, or 0 for spacelike, timelike, or lightlike. All familiar to me. H refers to the object V.V as the "**gorm**" of V. the "norm" might be $\sqrt{\text{V.V}}$ or $\sqrt{\text{-V.V}}$. In my tensor doc I mentioned the lack of name for this entity V.V, with reference to Messiah. Fine.

2.16 Reverse

Take clif C, reverse the ordering in all terms, and that gives you C~. Fine.

2.17 Gorm

In all generality, the gorm of any clif C is formed by multiplying C by the reverse of C, and keeping the scalar part of the product. That is:

$$\text{gorm}(C) := \langle C^{\sim}\, C \rangle_0 \tag{31}$$

For example, if $C = a + b\,\gamma_1 + c\,\gamma_2 + d\,\gamma_1\,\gamma_2$, then the gorm of C is $a^2 + b^2 + c^2 + d^2$ ... assuming γ_1 and γ_2 are spacelike basis vectors, as discussed in section 2.18. You can almost

Try it,

$(a+b\gamma_1 + c\gamma_2 + d\gamma_1\gamma_2)(a+b\gamma_1 + c\gamma_2 + d\gamma_2\gamma_1)$

$= a^2 + b^2\,\gamma_1^2 + \ldots$

Basis vectors I guess means $\gamma_1^2 = 1$, Do we have $\gamma_1\gamma_2 = -\gamma_2\gamma_1$? I don't see that, so the above claim does not fly with me. But, in PQ of vectors, we know there are only two pieces. Thuss we write

$$PQ = P \bullet Q + P \wedge Q$$
$$\quad 0 \qquad 2 \qquad\qquad \text{grade}$$

So OK, if γ_1 and γ_2 are orthogonal basis vectors, then $\gamma_1 \bullet \gamma_2 = 0$, and $\gamma_1\gamma_2 = \gamma_1 \wedge \gamma_2$ and we know that for vectors we always have $a \wedge b = -b \wedge a$. Therefore yes, we have $\gamma_1\gamma_2 = -\gamma_2\gamma_1$.

2.18 Basis sets

Well here is a strange section:

In Minkowski spacetime, any such basis will have the following properties:

$$\gamma_0 \gamma_0 = -1 \tag{32}$$

$$\gamma_1 \gamma_1 = \gamma_2 \gamma_2 = \gamma_3 \gamma_3 = +1 \tag{33}$$

and

$$\gamma_i \gamma_j = -\gamma_j \gamma_i \quad \text{for all } i \neq j \tag{34}$$

In this basis set, γ_0 is the timelike unit vector, while γ_1, γ_2, and γ_3 are the spacelike unit vectors.

Here we do have γ_i and γ_j perp, so yes, then you get $\gamma_1\gamma_2 = -\gamma_2\gamma_1$. I guess he was assuming this situation in the previous section! Fine. He sort of assumes the reader knows about relativity, but he has avoided using the word "metric" or "metric tensor". So far.

Confusion: If I think of the γ_i as basis vectors for the 1-blade space, then they are like the e_i of Suter's pdf. But there it seems that $e_{ij} \equiv e_i e_j = e_i \wedge e_j$ and then $e_{11} = e_1 e_1 = e_1 \wedge e_1 = 0$. I think Denker may have messed things up here! He never really says what his γ_i objects are. He really first defines them as orthonormal basis on page 16. Let's try his multiplication there if we assume $\gamma_i = e_i$

$A = \gamma_1 \wedge \gamma_2 \equiv \gamma_{12}$
$B = (\gamma_2+\gamma_3) \wedge (\gamma_4+\gamma_1) = \gamma_{24} + \gamma_{34} + \gamma_{21} + \gamma_{31}$

$AB = \gamma_{12} \wedge (\gamma_{24} + \gamma_{34} + \gamma_{21} + \gamma_{31}) = 0 + \gamma_{12}\gamma_{34} + 0 = 0 = \gamma_{1234}$

This is not the result he gives. A similar confusion is that he says $\gamma_1^2 = 1$ whereas Suter says $e_i e_i = 0$

2.19 Components

Here he treats the γ_i for i = 0,1,2,3 as I might treat u_i as basis vectors with the BD metric tensor. There seems nothing really new here. You recover the usual p.q dot product in terms of components. He puts component subscripts up, but does not say "contravariant". For example,

$$V = V^0 \gamma_0 + V^1 \gamma_1 + V^2 \gamma_2 + V^3 \gamma_3 \tag{36}$$

$$P \cdot Q = -P^0 Q^0 + P^1 Q^1 + P^2 Q^2 + P^3 Q^3 \tag{37}$$

Comment: So OK, in Minkowski 4-space (d=4), we have a set of basis vectors γ_μ and then we can express any vector as a lincomb of these basis vectors. There is probably also some kind of basis for the world of bivectors, and a basis for trivectors. What are these bases?

2.20 Dimensions; Number of Components

Mysterious table which he fails to explain.

$$\begin{array}{ccccccccc} & & 1s & & 1v & & & & D=1 \\ & 1s & & 2v & & 1b & & & D=2 \\ 1s & & 3v & & 3b & & 1t & & D=3 \\ 1s & & 4v & & 6b & & 4t & 1q & D=4 \end{array} \tag{38}$$

I think the D row labels are dimensions of underlying space. For R^3 we have a scalar having one component (1s). So in the pairs, second letter means s=scalar, b=bivector, t=trivector, q=quadvector. The number before the letter is the number of components this object has.

D= 3 and R^3: 1s = you have the usual single scalar
3v = you have a vector with the usual 3 components
3b = you have a three different 2D patches available
1t = you have only one possible cubic 3-blade object

The claim is that the picture has binomial coefficients going across. Pascal's Triangle. Total components on each row turns out to be 2^D. Get that from binomial expansion as $(1+1)^D = 1^3 + 31^2 1 + ...$

He comments that the spatial dimension D can be anything, and OK to do string theory with 26 dimensions. Clifford algebra does not care.

Comment: Author has used up 21 of his 35 pages doing this Chapter 2 overview business.

3. The Angular Momentum World

So he is staying with his physics leanings. All clear to me, but he is using ^ instead of cross products. Perhaps he is saying that **a** ^ **b** and **a** x **b** are the same for R^3 when clifs are vectors. H no doubt comments on this in his intro which I have so far skipped. Perhaps he was planning on expanding this section, it is just half a page.

4 Contractions : Generalizations of the Dot Product

Here he just mentions that you can talk about a variety of dot-like products, and they are all defined just in terms of the grade component of a clif product (a geometric product). Here is his table.

If $\widetilde{\ }$

	A	$= \langle A \rangle_r$	(homogeneous of grade=r)	
	B	$= \langle B \rangle_s$	(homogeneous of grade=s)	
Then				
	$A \lrcorner B$	$:= \langle AB \rangle_{s-r}$	(left contraction)	(42a)
	$A \llcorner B$	$:= \langle AB \rangle_{r-s}$	(right contraction)	(42b)
	$A \circ B$	$:= \langle AB \rangle_0$	(scalar product)	(42c)
	$A \bullet B$	$:= \langle AB \rangle_{\lvert s-r \rvert}$	(dot product)	(42d)
	$A \bullet_H B$	$:= \langle AB \rangle_{\lvert s-r \rvert}$	provided $r > 0$ and $s > 0$ (Hestenes inner product)	(42e)

Fine by me, nice to at least mention them in passing.

Loosely speaking, these all try to capture the idea that the contraction is the lowest-grade piece of the geometric product. The Hestenes inner product is almost never a good idea;

So this was just a tiny section too.

5 Hodge Dual and Cross Products [23]

The Hodge dual is a linear operator. In d dimensions, it maps a blade of grade g into a blade of $d-g$ and vice versa. I call this the the grade-flipping property. Two particularly-common

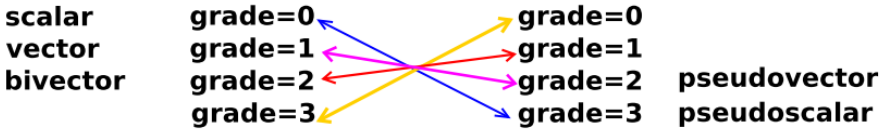


Figure 5: Hodge Dual in $d = 3$

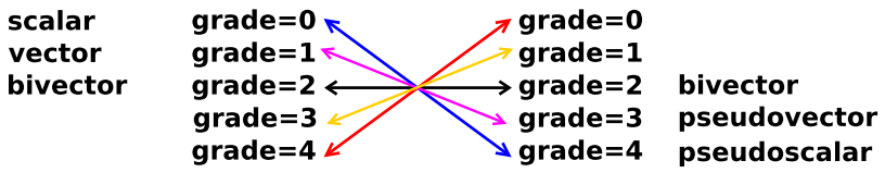


Figure 6: Hodge Dual in $d = 4$

This is roughly in agreement with the * Hodge discussion in Sjamaar and elsewhere.

Next comments are:

V=vector	C=blade grade g	V^C = grade g+1	V^ = grade raiser
V=vector	D=blade grade g	V.D = grade g-1	V. = grade lowerer

So a vector raises or lowers the grade for a wedge or dot product.

Finally things are getting thicker here and I am going to have to ponder more. On page 24 of 35.

I know about Hodge in the sense that *dx = dydz and I guess *2 = dxdydz and *dxdydz = 1 more or less. So adjust this to map blades instead of differentials. A blade is a pure-grade clif.

OK. start with blade A,B each of grade k. The Hodge X will have grade d-k so we know that much.
What do you know about A^X? Add grades so it has grade d and is thus called (reason unknown) a pseudoscalar as chart above shows. Meanwhile A.B has grade 0.

He says now to find some "unit pseudoscalar i". Perhaps that means $i^2 = 1$.
He says find a basis γ_i for i = 1 to d, seems OK to do that.

Define $i \equiv \gamma_1\gamma_2....\gamma_d$ where this is geometric multiplication about which I know little, PQ.
Well I know PQ = P.Q + P^Q so I guess of basis is orthogonal, $\gamma_1\gamma_2 = \gamma_1\wedge\gamma_2$. Then we must really have $i = \gamma_1\wedge\gamma_2....\wedge\gamma_d$. The basis are vectors so grade 1, so then i has grade d which is correct. It is in some sense a "unit grade d blade" I suspect.

Now he claims

A^X = A.B i

All I really know from above is that A^X has grade d, and A.B has grade 0 and i has grade d, so how does he justify the above equation? Each side is perhaps a blade of grade d. But one could be a scalar multiple of the other? How many grade d blades are there? Probably only 1, so only have to confirm the constant. A has grade k. Do I know some simple grade k blade? Perhaps $A = \gamma_1\wedge\gamma_2....\wedge\gamma_k = \gamma_1\gamma_2....\gamma_k$. I just don't know enough to explain the above equation. Denker has let me down.

Wiki says this which I am more familiar with

$$\star(e_1 \wedge e_2 \wedge \cdots \wedge e_k) = e_{k+1} \wedge e_{k+2} \wedge \cdots \wedge e_n.$$

How do you expand X in basis vectors knowing it has grade d-k? I don't know.

So just assume the above for now. If the above is valid for all A, then X is the Hodge dual of B. I will write this as X = $*_i$ B, but he uses an awful notation I can't even type. For a given i, the Hodge dual is unique, he says.

Try his example: B = 7, a scalar, what is *B. He claims *B = 7i which is indeed a d=0 object. Let's check so see if A^X = A.B i: If B = 7, then I suppose A.B = (1/2)(AB+BA) = 7A. So then we have that A^X = A7i for all A. Somehow the solution he claims is X = 7i. I need more exercises!

Example: In $d = 3$, suppose $B = 5\gamma_1\gamma_2$ (i.e. a bivector in the xy plane). Then $B\S_i = 5\gamma_3 = [0, 0, 5]$ (i.e. a vector in the z direction). This assumes $i = \gamma_1\gamma_2\gamma_3$.

So in this case have to show that A^X = A.[5$\gamma_1\gamma_2$] i for all A. I have no idea. I can't even verify the above example! I have no A's to test with! Have never met an A other than a scalar.

5.2 Remarks : Subspace Freedom, Or Not

Chatter about writing * as $*_i$ maintains "subspace freedom". I do not grok.

5.3 Recipe for Replacing Cross Products

In this subsection, we restrict attention to $d = 3$. We choose our preferred unit pseudoscalar in the obvious way, namely $i = \gamma_1\gamma_2\gamma_3$.

With this standard choice of "i", he claims that, assuming a and b are vector clifs

a x **b** = *(a^b)

Note that a^b has grade 2, and then *(a^b) has grade d-2 = 3-2 = 1 which is in fact a vector. So the above equation is true at least in the grade matching sense. But I don't know how to strictly verify this. Then we get this next claim

$$\begin{aligned} A \times B \cdot C &= A \wedge B \wedge C \\ |A \times B \cdot C| &= |A \wedge B \wedge C| \end{aligned}$$

6 Pedagogical Remarks

6.1 Visualizing Bivectors

Spinning top, but I don't get it. I never had trouble adding angular momentum vectors.

6.2 Symmetry

Cross product implies presence of chirality, clifs do not, advantage for modeling non-chiral physics.

6.4 Geometric Approach versus Components

This is a sales pitch. He avoided a discussion of clifs having components and expansions on a basis, and I really did not like that. He wants to do "New Math" where basis is downplayed. The vector in 3D you can easily identify without talking about basis. But of course if you want to compute anything (heaven forbid for a string person) you will need a basis. So I am not a disciple here. Wonder if this relates to general relativity and Kushar's remarks to me at the Jim fest?

7 Clifford Algebra Desk Calculator

This program supposedly calculates anything you want in the clif world. Language is not stated, again because that would mean selecting a basis.

Comments: Well, this doc DID introduce the basic ideas, but I would have preferred a more traditional approach. So I will hunt some more, I did not know I was getting a proselytizer as a presenter when I downloaded this pdf. I should have suspected on looking at his web hits.

Comment: OK, I now want to see a more conventional Clif presentation which does use basis vectors and linear combinations and all that. I spent some time looking for Clif pdf's and found few, but most tend to be overly fancy for me..