Jim Scattering Notes PhL 1.21.05

I put Jim's notes in a binder to try to keep them in some reasonable sorting order. I then tried to review these notes just to see what they are about. They are quite inscrutable, but here are some of the topics he touches upon:

- 2D scattering from a potential V(r) and later from "potential well"
- $J_\upsilon(..)$ Bessel functions appear everywhere, and sometimes their zeros $x_{\upsilon n}$
- some pages of FORTRAN code, some talk of doing FFT

Nothing here makes any sense to me, it is just Jim scribbling as he ponders details of some problem he is thinking about, but of course I don't know at all what he is doing. When I assembled the binder, I put the notes into "sections" thinking they might be on different topics, but pretty much the whole thing is all mushed together. Probably the sections are just in my imagination.

I have put my comments on his scribbles at the end of this document

Now, what OTHER materials did he give me?

(1) A Gautschi paper which talks about the susceptibility of polynomials to having numerical problems, the so called "conditioning" issue. Things have a "**condition number**". Large number means ill conditioned. He has some examples involving working with ortho polys and even mentions half-range Hermites. Perhaps the iteration schemes to get coefficients have conditioning problems.

(2) A copy of Jim's half range Hermite paper (Jim 3). This is a 7 page shorter version of the 9 page paper I have, I can see that there are differences, but I don't know which is newer.

(3) A preprint copy of the Jim 2 Bessel paper.

(4) One issue of Computing in Science and Engineering Nov 1999, a theme issue on physicists getting into the world of financial calculations. Aha but now I see: at the end Jim has an article on ortho polys. This is in fact a version of the Jim 1 paper with some extra stuff added, a graph for his Laplace thing. The introduction is completely new here. **He credit's Wilf** for the Jacobi method. Jim comments that he taught the course for 30 years, but only learned of Wilf "a few years ago" relative to 1999. References are A&S, Wilf, and Recipes. He claims that he is not alone in learning only lately of this Wilf method. So no need to look up weights in a table.

(5) A **floppy** with some fortran code on it, I copied all into my download area. No comments in the code, seems related to the GQ papers though.

(6) The Watcom **compiler** for FORTRAN. I think I have this all installed.

(7) A **book**: Newton, *Scattering Theory of Waves and Particles*, 1966. I see this has a "new" Dover publication for $35 if I ever need it, an expanded 1982 edition.

I see now that Jim has marked a section on "**the inverse scattering problem**" . Here I see all those things he has been scribbling: The D_1 operator, and g(r,r') and K(r,r'), the $L_{\ell\ell'}$ thing. This is a fairly short chapter, and now at least I know more the area Jim is fiddling in.

Status: Jim is interested in this inverse scattering problem and it does relate to Techniscan. I am not sure this topic is directly connected to his interest in GQ. He wonders about the following problem: scattering coming in from the side to a rotating uniform infinitely long cylinder. Things this is not a solved problem. But why is this useful? Claims you will get an up and down Doppler shift in the reflections. I do recall that there is some problem between Jim and Steve Johnson, perhaps on point of view of scattering and doing numerical work on same.

Jim is not easy to work with because he does not measure other people's comprehension as he talks, I can imagine this was a huge problem in his teaching career. I think Fred did the "presentation" of things in the past, and Jim did "the calculations".

I think I will politely decline to get involved here. Thomson has some paying Verilog work coming up anyway, so perhaps this is a good time to return all those materials. I think I did do diligence with Jim by reading and commenting on all his GQ papers. They were interesting, and caused me to review large sections of math which I am glad I did.

Jim's Scribbles

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Section 1

page 1

He starts with a Bessel function expansion of exp(i$\mathbf{k_0 \bullet p}$). Coordinates are ρ and θ, and he is talking 2D scattering, so I think of a scatterer at the origin in the x,y plane with ρ and θ as the coords.

Then ψ is written as this plane wave plus some scattering and we have 1/r on the probability I guess since 2D. The scattered wave is expanded in first kind Hankels H with amplitudes A_n. We are scattering off a "potential". Some kind of matching of BC's, but unclear what the potential is. Incident direction is $\mathbf{k_0}$ and outbound scattering is at $\mathbf{k}$, hence f($\mathbf{k,k_0}$).

page 2

Here we are messing with amplitudes A_n and B_n and C_n which show up in an expression for ψ. Bottom of page 2 gives an expression for f($\mathbf{k,k_0}$) in terms of phase shifts δ_n . I have this all somewhere, but have no idea where -- my notes will be in 3D.

page 3

Rewrites f($\mathbf{k,k_0}$) and then things come to a halt.

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Section 2

page 1

We have what looks like a completeness relation for Bessel J_n with x_{nm} being I think the zeros. All very mysterious, disconnected from everything else.

page 2

Talking here about the recursion relation for $J_\upsilon(x)$, does not go anywhere.

page 3

Talking about $J_2(x_{01})$ and similar simple things, results to 3 decimal places.

page 4

Strange equations involving expansion of cos(z cosϕ). Very strange.

page 5

He is trying here to extend the normal Jacobi range (-1,1) to (a,b), just a few scribbles.

page 6

Jim's retirement questions to Scott Drumond about roth IRA and when to start soc sec.

page 7

scribbles on a simple integral

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Section 3

page 1

I. He writes the amplitude on a **drum head**. This confirms that $x_{n\ell}$ is a zero, so amplitude will be zero at the rim at r = a. Radial modes are then the ℓ index, first index is the J_n index. II. Some kind of coefficients a_ℓ suddenly appear, a function $g_n(r)$ appears, maybe total radial solution. III. Orthogonality of J_n with itself but at different zeros. Maybe this is related to my $|x_i>$ space. IV. Quadrature he says. I guess we are going to try to integrate something soon.

page 2

General form for a potential V(**r**) in 2D in terms of r,θ expanded in $J_n(kr)e^{-n\theta}$ with $A_n(k)$ coeffs. The n is the Fourier conjugate of θ, and the k that of r. I presume $\phi(r)$ is going to be some scattering amplitude off this potential, big messy double integral over r and θ with J_m and Hankel(1)'s appearing. Rewrites

double integral several times. I don't know where this J expansion is coming from, it must be the expansion of the Green's function.

page 3

Title is "**Rytov** in Polar Coordinates". Born, Rytov and Eikonal are various scattering approximations. Here we have one equation with ∇^2 and ∇ acting on the total wave function of incident plane wave plus scattered ϕ where n is some integer. We have a propagator style integral for $\phi(r)$, Green's is $G(k-k_0)$ but we go back and forth k-space and r-space by Fourier. Specialize to cylinder geometry at page bottom. Ah yes, remember that in 2D you get that Hankel function.

page 4

Comment about FFT evaluation of sum $S(\theta) = \Sigma\, A_n\, e^{in\theta}$, go into partial waves S_ℓ, goofing around.

page 5

Rewriting $\phi(r)$ as Σ_n of Hankel $H_n^{(1)}$ and new thing T_n which is then a radial integral. Some fiddlings on how to handle the ℓ sum that also appears.

page 6

Title "2D scattering". I. General form of scattered ψ, says "compute with FFT". II. Comments on unitarity, effect on wave function.

page 7

Suddenly we are at item V: relates f_n to a_n , more goofing around. then VI: some expansion of Si with those a_n now appearing.

page 8

Study of some θ integrals, just fiddling more.

page 9

A one page FORTRAN program with no comments. No, first page of a longer listing, file RWAVE2D.FOR. Who knows?

page 10

Fiddling with $J_\ell(r)$ and some K(r,r') and $A_{\ell,\ell'}$. I presume this is the same K(r,r') as in Newton.

page 11

more scrawling with J_ℓ and this K thing.

page 12

still more, begins with the Newton D_1 differential operator in r. Now we see Newton's g and K functions floating around

page 13

continue half page with this D_1 . Peters out. Recall that the thing at the top should be 0 according to Newton.

page 14

More fiddling with K(r,r') and this g(r,r') and D_1 .

page 15

now we are talking some ψ_ℓ thing and ψ_ℓ' .

page 16

Title is "square well potential". More J_ℓ fiddling.

page 17 through end of this section

continues on square well

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Section 4.

page 1

We seem to be continuing on with ψ_ℓ' and K(r,r') and some $A_{\ell\ell'}$

page 2

Statement of the integral equation involving g(r,r') and k(r,r'), as appears in Newton. Apply our D_1 to it (defined on page 12 of previous section).

page 3

more of the same. Trying an expansion for k(r,r').

page 4

Here is the definition of $L_{\ell\ell'}$ as integral of J_ℓ against $J_{\ell'}$

page 5

Has an ODE relating K(r,r') to V(r'), the scattering potential I presume. Then suddenly switches to FFT and FORTRAN stuff, as if writing out a program.

page 6

more on this little program. Just writing out values of a Bessel function I think.

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Section 5.

page 1

Here we have the $\delta(r-r')$ completeness for J_n with Σ_n. Those zeros are showing up again. I guess you can regard this as the spectral sum with ρ_1 .

page 2

Now we are back to K(r,r') as Σ_ℓ of c_ℓ and J_ℓ and ψ_ℓ. Define operator D_0 but looks like same D_1 as we had earlier. Writes ODE for J_ℓ . Repeat of earlier symbols here.

page 3

Computing a certain integral I_n of a J_n expression.

page 4

writes a recursion for this integral I_n.

page 5

Two page fortran program of Nick Stoke which finds roots and weights for half-Hermites. Seems out of place.

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Section 6 3 stapled together pages

I think Jim likes to make a little outline with Roman numerals I, II, III ... to plan what he will do. Here we have I: expand $\psi(r,\theta)$ in partial waves n with Hankel-1 functions of r. II. Match BC's. III Use symmetry, and now talking about discrete θ_i so perhaps doing numerical here. Writes S_i and D_i as "sum and difference" of ψ_i and ψ_{-i} and expands these. IV: apply unitarity, more messing. Then VII and we are playing with least squares fit .

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