**Meaning of saying: A is an open subset of B.** PhL 3.1.16

1. First, go look at the definition of an **open set** given as defined in Topology World:

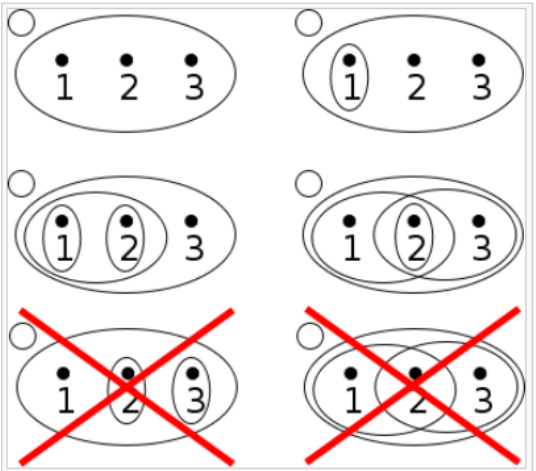
[ I already have some notes in "compactness.doc" ]

### Open set definition [ edit ]

A *topological space* is an ordered pair $(X, \tau)$, where $X$ is a set and $\tau$ is a collection of subsets of $X$, satisfying the following axioms:[4]

1. The empty set and $X$ itself belong to $\tau$.
2. Any (finite or infinite) union of members of $\tau$ still belongs to $\tau$.
3. The intersection of any finite number of members of $\tau$ still belongs to $\tau$.

The elements of $\tau$ are called **open sets** and the collection $\tau$ is called a **topology** on $X$.




Four examples and two non-examples of topologies on the three-point set {1,2,3}. The bottom-left example is not a topology because the union of {2} and {3} [i.e. {2,3}] is missing; the bottom-right example is not a topology because the intersection of {1,2} and {2,3} [i.e. {2}], is missing.

My first question: What is the connection between this and my usual notion of an open set in terms of an open ball around a point in the set being in the set?

Well, the above topo definition can be applied to spaces which have no distance (no metric) and can be used to sort of replace the idea of distance, examples being those shown above where there is no meaning of distance.

I presume that when you do have a metric, you can show that the open ball definition of open set implies the topo open sense as well. Is this a trivial thing I can show right now?

The topology that consists of ALL subsets of a set is called the **power set**.

Consider a set of sets of R where the sets are "open neighborhoods" around all elements of R, where open is in the usual metric sense, plus the null set. Does this set of sets form a topology? Rule 1 is OK since the full R axis is a neighborhood of any point in R. If A and B are two neighborhoods, their intersection is either null, which is OK, or it is an open neighborhood around any point in the intersection. Union is OK too, so the answer is yes. This is called the **metric topology** on a metric space, and if you also are a vector space, then it is called the **standard topology**.

Now consider U being [0,1] or [0,1). The problem here is that the endpoint 0 fails to have any neighborhoods! Thus you cannot put open balls around all points, so you cannot construct the metric topology on [0,1). Remember that you need to be able to have an open ball around all points of your set.

2. Now look at wiki Open-set page:

**"Open" is defined relative to a particular topology** [ edit ]

Whether a set is open depends on the topology under consideration. Having opted for greater brevity over greater clarity, we refer to a set $X$ endowed with a topology $T$ as "the topological space $X$" rather than "the topological space $(X, T)$", despite the fact that all the topological data is contained in $T$. If there are two topologies on the same set, a set $U$ that is open in the first topology might fail to be open in the second topology. For example, if $X$ is any topological space and $Y$ is any subset of $X$, the set $Y$ can be given its own topology (called the 'subspace topology') defined by "a set $U$ is open in the subspace topology on $Y$ if and only if $U$ is the intersection of $Y$ with an open set from the original topology on $X$." This potentially introduces new open sets: if $V$ is open in the original topology on $X$, but $V \cap Y$ isn't, then $V \cap Y$ is open in the subspace topology on $Y$ but not in the original topology on $X$.

As a concrete example of this, if $U$ is defined as the set of rational numbers in the interval $(0, 1)$, then $U$ is an open subset of the rational numbers, but not of the real numbers. This is because when the surrounding space is the rational numbers, for every point $x$ in $U$, there exists a positive number $a$ such that all *rational* points within distance $a$ of $x$ are also in $U$. On the other hand, when the surrounding space is the reals, then for every point $x$ in $U$ there is *no* positive $a$ such that all *real* points within distance $a$ of $x$ are in $U$ (since $U$ contains no non-rational numbers).

I am pretty sure this idea is the same as Buck's idea of **"open relative to"** on page 31.

The subsets of all rational numbers form open sets relative to the rationals, but such subsets are not open relative to the reals. The reason is that any ball around a rational that includes only the nearby rationals does not include any reals so every rational number is on a boundary relative to the reals!

**Application**: Consider Sjamaar's φ(U) = V where φ is 1-to-1 so if U is open, then V is open. But suppose U is a small open patch in $R^2$ so that V is an open patch of dimension 2 which lies on some 2D surface in $R^3$. The set V is open relative to that 2D surface on which it lies, but V is not open relative to $R^3$ ! On page 36 bottom Sjamaar says his k-form is defined on an open subset V of $R^m$, then he has famous picture on the bottom of page 37. I think V as an n-dimensional surface (the map of U) is open in an n-dimensional sense, but not relative to $R^m$ where m > n. It would then be correct to say that V was an open subset in $R^m$ but it is not open relative to $R^m$ , it is open relative to the n-dimensional manifold.

Think of V containing no boundary points on the manifold so it is open on the manifold. Fine. But you cannot put $R^m$ open balls around points on the manifold which balls lie in the manifold, so I would say that manifold M is not open relative to $R^m$.